A MODEL TO SOLVE EN ROUTE AIR TRAFFIC FLOW MANAGEMENT PROBLEM:

A TEMPORAL AND SPATIAL CASE

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ABSTRACT: *The paper considers the air traffic Flow Management Problem (FMP) for an airspace case. In this case temporal and alternative spatial solutions are allowed, i.e. both ground holds and re−routing (as well as combination of the two). The formulation of a general model is proposed and developed into a BIP−based model. Solutions are obtained using LP supplemented by a heuristic method (developed earlier). Some experimental results are shown as well.*

KEY WORDS: *Air Traffic Control, Air Traffic Flow Management.*

1. INTRODUCTION

The FMP (Air Traffic Flow Management Problem) can be generally defined as the optimum assignment of limited Air Traffic System resources (airspace, sectors, airports etc.) to Air Traffic System clients (flights) in situations when demand exceeds capacity.

The problem statement, including a real life system, modeling problems and solutions, as the authors see them, were presented in detail by Tosic and Babic (1995). Some solutions together with discussion of the proposed models were presented by Tosic et al. (1995). This paper will explain some of the research results which are an extension of the authors' previous work.

The extension includes a spatial solution to the problem, together with a temporal one.

2. TEMPORAL AND SPATIAL SOLUTION

Temporal solution is based on the assumption that each flight will have to follow a single (optimum, desired, chosen) 3D trajectory through the system network.

This implies that the only way to solve the problem of overload in some nodes, which appear when demand exceeds capacity, is to delay some of the flights (the overload period, of course, must be followed by an underload period). 3D trajectory is unchanged, but 4D will be changed for some flights.

If attempting to bring the model closer to the way a real system operates, the fact that a flight which has been assigned (or is threatened to be assigned) to ground hold because of en route congestion might be offered or ask for an alternate route. This new route will be longer then the original one, but the shorter delay (at destination), as a result of no ground hold or shorter than if the original route had been chosen, might be more acceptable to the carrier. This introduces spatial as well as temporal solutions to the FMP, generally defined as follows:

The system supplying service to customers consists of a finite number of elements. Each element can handle several customers simultaneously but has a finite capacity which may vary sin time. Individual customers demand service from one (rarely) or more elements sequentially. Each customer has his first (optimum) choice sequence, but also his alternate choice(s). The delay cost functions for each customer's first and alternate choice(s) are known. Customers choices are initially independent.

The problem appears if total demand exceeds the capacity of some elements at certain periods. The assumptions are:

> That an overload is followed by a lower−demand−than−capacity period when the overflow of demand from the previous period can be served (completely if only temporal solution is allowed), and
> That during such the overload period, each flight originally routed through a bottleneck node has an alternate, longer route, passing through nodes which are not bottlenecks.

The question is:

> How to manage demand so as to minimize delay cost to customers, or more precisely, which customers should be delayed on departure (ground hold) and for how long, going on the original routes, and which ones should be given alternate routes (rerouted) with possible ground hold (length of which should also be determined), in order to yield minimum total cost?

The problem of the delay cost incurred both on the ground, in the air, and combined, will not be discussed here. Some very important aspects of delay cost functions have been discussed by Tosic et al. (1995). In addition to this, however, it should be emphasized that in the case of a spatial solution, the problem of deciding what a delay cost function is and how carriers perceive it, becomes even trickier. It will be assumed here that these cost functions are known.

3. GENERAL MODEL

Notation:

F	−	set of flights indexes
F_a	−	set of flights indexes from F that have alternative routes ($F_a \subset F$)
A	−	set of route indexes, alternatives for flights from F_a, considered as additional flights
N	−	total number of elements
T	−	total number of time periods
$D=\lVert d_{ijt} \rVert$	−	d_{ijt} is 1 if flight i demands service at element j during period t, and 0 otherwise ($i \in F \cup A$, $j=\overline{1,N}$ 1, $t=\overline{1,T}$ 2)
Q_{jt}	−	total capacity of element j at period t ($j=\overline{1,N}$ 3, $t=\overline{1,T}$ 4)
k_i^*	−	maximum allowed delay of flight i ($i \in F$) when assigned to original route

k_a^*	−	maximum allowed delay of flight i ($i \in F_a$) when assigned to its alternative route a ($a \in A$)
$c_i'(k)$	−	total ground delay cost of flight i delayed for permitted number of k periods ($i \in F$)
c_i''	−	total additional constant delay cost of flight i assigned to the alternative route ($i \in F_a$)
$x_{i,k}$	−	equals 1 if flight i is delayed for permitted number of k periods, and 0 otherwise ($i \in F \cup A$).

Now the discrete model can be formulated as:

$$(\min) \sum_{i \in F} \sum_{k=0}^{k_i'} c_i'(k) \cdot x_{ik} + \sum_{(i,a)} \sum_{k=0}^{k_a'} \left(c_i'(k) + c_i'' \right) \cdot x_{ak} \quad (1)$$

subject to:

$$\sum_{k=0}^{k_i'} x_{ik} = 1, \quad i \in F \setminus F_a \quad (2)$$

$$\sum_{k=0}^{k_i'} x_{ik} + \sum_{k=0}^{k_a'} x_{ak} = 1, \quad \forall (i,a) \quad (3)$$

$$\sum_{i \in F \cup A} \sum_{k \geq k} d_{ij,t-k} \cdot x_{ik} \leq Q_{jt}, \quad j = 1,\ldots,N \;\; t = 1,\ldots T \quad (4)$$

$$x_{ik} \in \{0,1\}$$

The first sum in (1) represents total ground delay costs of flights assigned to original routes. The second sum expresses total ground delay costs and additional costs of flights assigned to alternative routes. This sum and the constraints (3) are established over all pairs (i, a), where a is the alternative route of flight i ($i \in F_a$, $a \in A$). The constraints (4) express the requirement that total demand should not exceed the capacity at any element and period. The sum in these constraints is realized over all allowed delays k less than period t.

The discrete model (1−4) is a problem of linear Binary Integer Programming (BIP). The BIP model expresses minimization of the total delay cost. Obviously, x_{ik}=1 for $i \in F$ means that flight i is assigned to the original route with delay k. In the case of $i \in A$, flight corresponding to the alternative route i is assigned to this route with delay k. The model as formulated here may not produce a feasible solution.

Some very important features of the proposed model are:
First, it covers all different cases of the FMP defined by objective functions (Tosic et. al. 1995).

Second, the model stays linear even when the delay cost is not linear, because the nonlinear dependence is expressed here through the objective function coefficients.

Third, it could be easily adapted to cover cases with more than one alternative route *a* for flights *i*, $i \in F_a$, considering these routes as additional flights and adding the corresponding sums to the left side of the constraints (3).

4. PROPOSED SOLUTION

The LP relaxation approach is used for solving the BIP model (1−4). LP relaxation of the BIP model is solved using a standard LP package. If the solution is integer, it is an optimal one and the value of the objective function (1) represents the minimum total delay cost. Otherwise, each flight *i* with value 1 for corresponding variable $x_{i,k}$ is served with delay *k*. Then all flights with non−integer values for some of the corresponding variables are served by applying one of the heuristic approaches described in Tosic et. al. (1995).

5. NUMERICAL EXAMPLE

In this section, results of one of the conducted experiments will be presented in order to illustrate the application of the developed model and proposed method of solution. It is similar to the numerical example used in Tosic et. al. (1995) were we had N=4 ATC sectors (North=N, East=E, South=S, West=W) and T=8 half−hour time periods with 162 flights in set F each intending to fly en−route through some of those four sectors. In this example we introduce two more ATC sectors, N' and E' (Figure 1) offering two alternative routings for the two most loaded air traffic flows, namely NE flow with 45 flights and EN flow with 26 flights.
Those flights are "offered" the following alternative routes: for NE flights, the alternative route is N'E' and for EN flights the alternative route is E'N'. Remaining flights of other traffic flows are not offered alternative routes. Therefore the initial total traffic load remains the same as in the example used in Tosic et. al. (1995), as well as sector capacities per period which were set equal to Q_{jt}=18 flights (j=1,2,3,4, i.e. j=N,E,S,W and t=1,2,...,8). Sectors N' and E' have capacities $Q_{5t}=Q_{6t}$=9 (t=1,2,...,8) because of the load produced by flights flying through those two sectors on their original route (flights not considered in this example), so those two capacity values should be understood as remaining sector capacities.

Alternative routes N'E' and E'N' are equal in length and take 1/3 of a half−hour time period (10 minutes) longer to fly than their respective original routes NE and EN. It is assumed that all flights originally in NE and EN flows will fly the same extra amount of time when flying on alternative routes for reasons of simplicity. For the same reason, we assume that the ratio of cost per unit time when the aircraft is in the air and when it is on the ground equals 4 (Andrews 1993), and this value was used for all flights. We used the same coefficients c_i as in the previous example, Tosic et. al. (1995) where the function of aircraft size given ranged from 0.1 for aircraft up to 15 seats to 4.0 for aircraft with 320 to 450 seats. We have therefore the following values for coefficients $c_i'(k)$ and c_i'' in objective function (1): $c_i'(k)=c_i \cdot k$ and $c_i''=4 \cdot 1/3 \cdot c_i=4/3 \cdot c_i$. Maximum permitted ground delay per flight i was set at k_i^*=4 half−hour time periods and was assumed equal for all flights. Then, maximum permitted delay for flights i on alternative routes a was calculated as $k_a^*=\text{int}(k_i^* - 1/3)=3$ and was also assumed equal for all flights assigned to alternative routes.

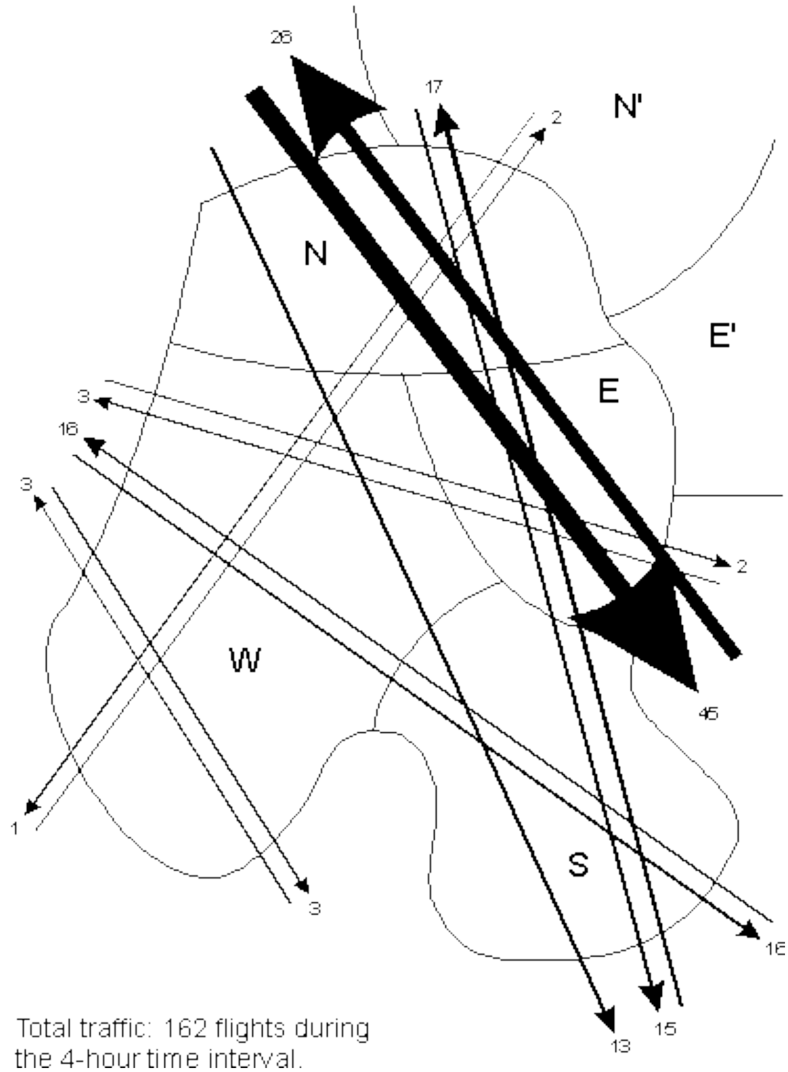

Figure 1: Example flow patterns

Using the proposed method of solution, we obtained the following results: 148 flights were scheduled on time flying their original routes, 5 flights had ground delays and were also assigned to their original routes with a total of 7 half−hour time periods of ground delay, while 9 flights were assigned to alternative routes without ground delay.

The results are shown in Figures 2 and 3, compared to the results of the case when only temporal solution was allowed.

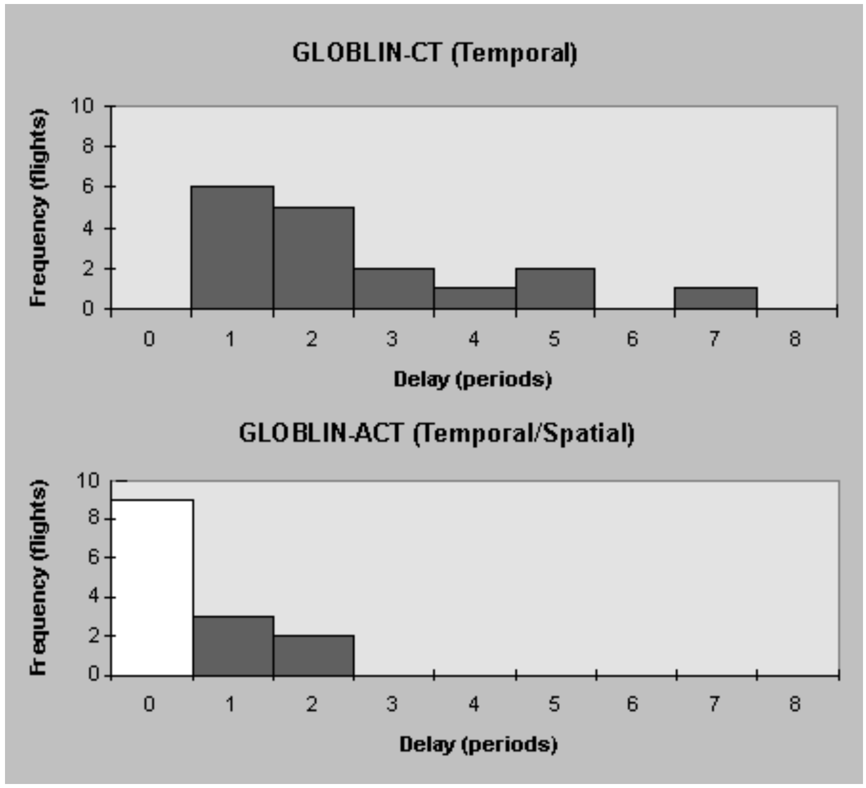

Figure 2: Distribution of ground delay for flights assigned to original and alternative routes

Figure 4 represents the state of capacity loads through time periods after solving our example FMP (shaded fields in the table indicate that traffic load has reached the sector capacity per time period).

This solution differs from the one obtained using only ground delays, as described in Tosic et. al. (1995) where we had 145 flights on schedule and 17 flights with different lengths of ground delay totaling 43 half−hour time periods of delay.

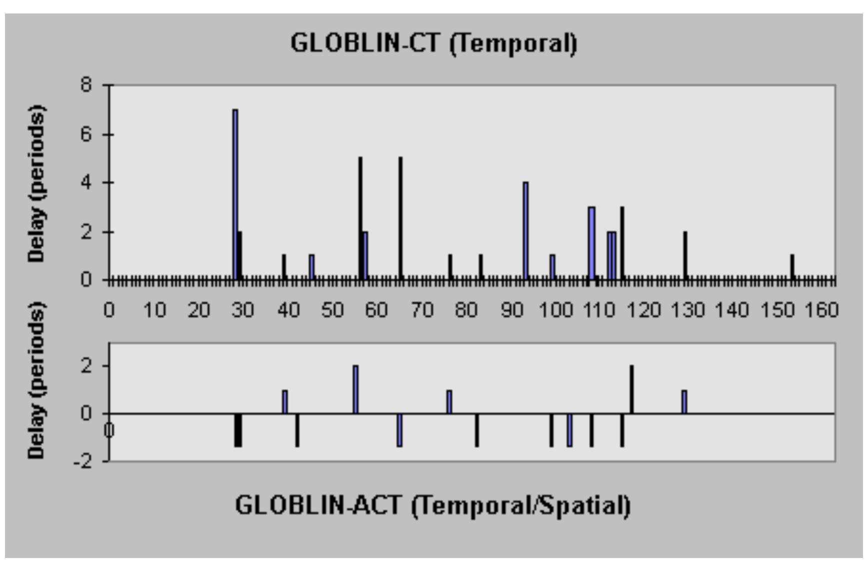

Figure 3: Per flight delay

GLO−CT	Period								
Sector	1	2	3	4	5	6	7	8	9
N	10	18	18	18	18	18	18	18	< 9 + 8
E	9	14	14	18	18	17	18	18	< 9 + 9
S	11	17	14	10	16	13	11	12	< 9 + 0
W	5	11	10	5	14	11	9	7	< 9 + 0

GLO−ACT	Period								
Sector	1	2	3	4	5	6	7	8	9
N	10	18	17	18	18	16	18	18	< 9 + 0
E	9	14	13	17	18	17	18	17	< 9 + 1
S	11	18	14	9	18	12	11	11	< 9 + 0
W	5	12	9	6	14	10	9	7	< 9 + 0
N’		3	2	1	2	4			
E’		2	2	1	4	5	1		

Figure 4: System load after solving the example problem

The value of objective function in a similar example described in Tosic et. al. (1995) was 31.6 which is much higher when compared with the value of 17.0 obtained with combined ground delay and alternative routing.

6. DISCUSSION AND CONCLUSIONS

The model offers a spatial solution for FMP combined with a temporal one. Generally speaking, this is a very realistic approach and should be available to the decision maker.

It is obvious that the solution gained by offering alternative routes beside simple ground hold is quite different from the above mentioned solution obtained by Tosic et. al. (1995). Here both total cost and total ground delay are less for flights on original routes. In this example, then it can be concluded that, under above described assumptions, it is worth offering alternative routes for heavy load traffic flows.

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