Ground-based Estimation of the Aircraft Mass, Adaptive vs. Least Squares Method

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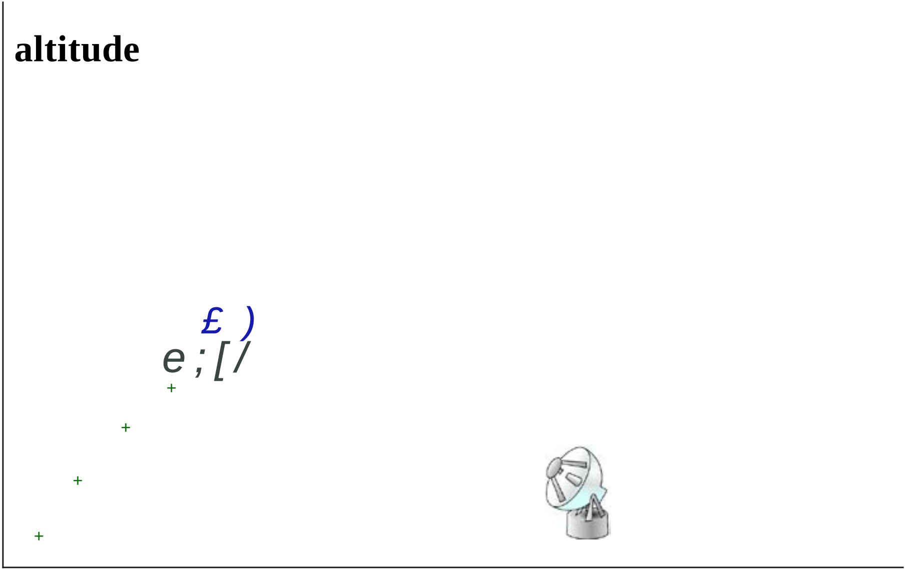
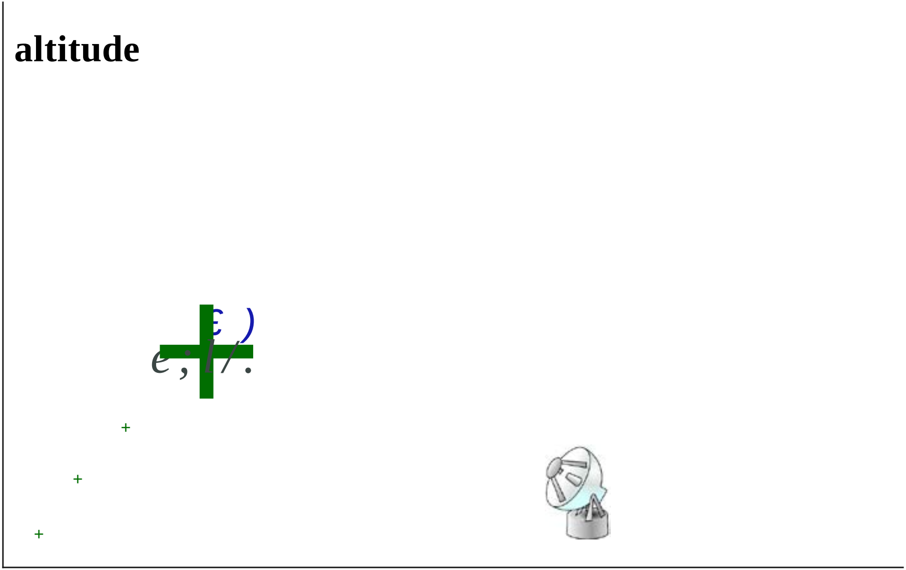

altitude

Introduction



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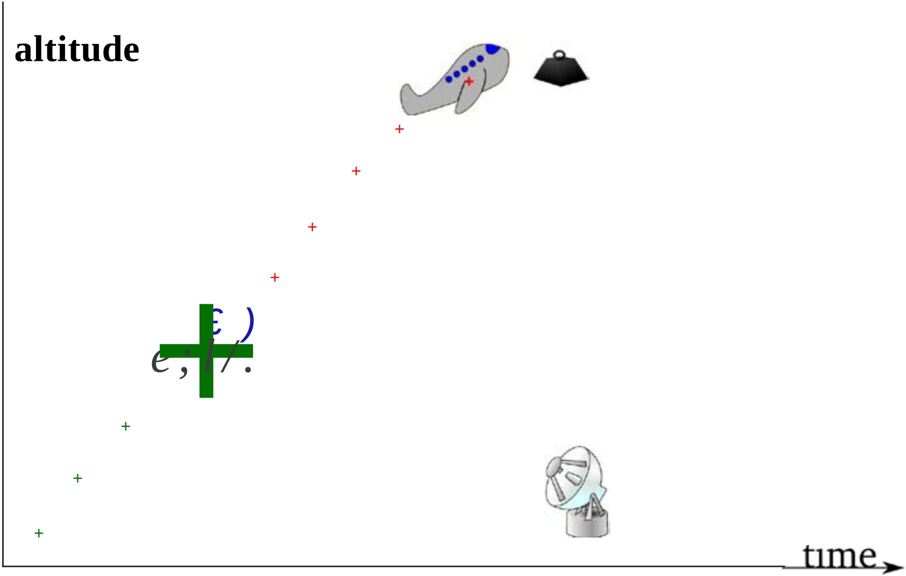

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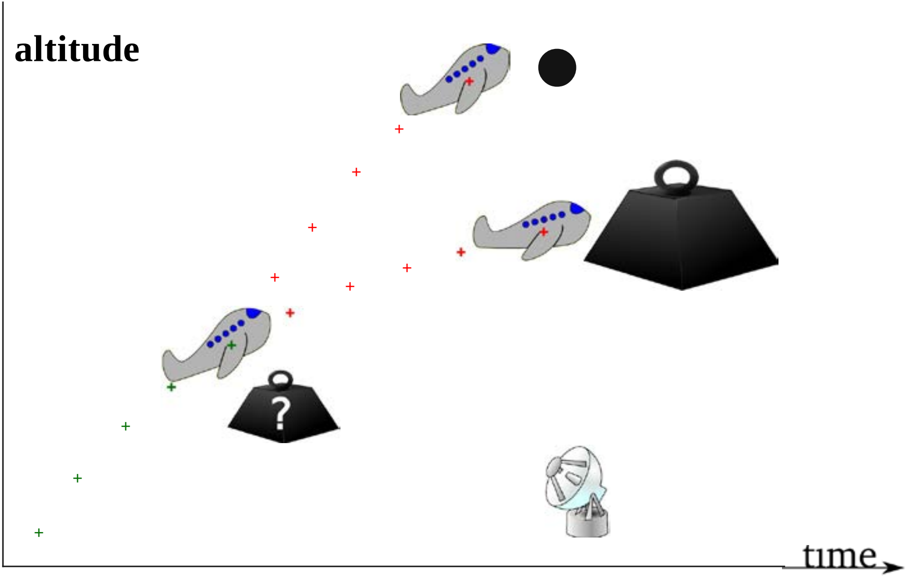


An *energy-rate* oriented approach

Newton's laws

$$\underbrace{\frac{1}{2}\frac{dv^2}{dt} + g\frac{dz}{dt}}_{energy\text{-}rate} = \underbrace{\frac{power(mass)}{mass}}_{f(mass)}$$

f is given by a physical model of the forces

Using past positions given by radar

- We compute the observed *energy-rate* from radar data
- We search a *mass* such that:

$$\text{observed } energy\text{-}rate = f(mass)$$

Objective

Previous work

- [Schultz et al., 2012] An adaptive method
  - Synthesized data
  - Without noise
- [Alligier et al., 2012] A least square method
  - Real data
  - No result on the mass estimation accuracy

In this work

- Synthesized data generated using BADA 3.9
- Gaussian noise added to the observed variables
- BADA 3.9 model of forces is used to estimate the mass
- Comparison of the mass estimation accuracies

1. Computing the $\frac{power}{mass}$ provided by BADA
2. The adaptive method [Schultz et al., 2012]
3. The least square method [Alligier et al., 2012]
4. Results

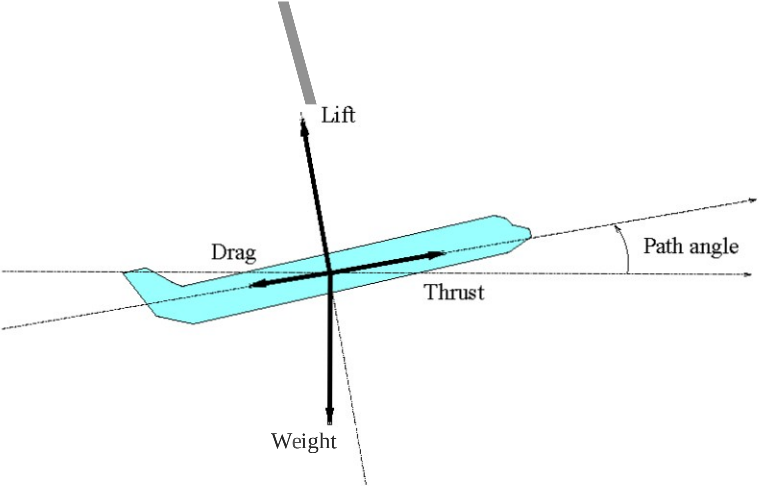
1 Computing the $\frac{power}{mass}$ provided by BADA

2 The adaptive method [Schultz et al., 2012]

3 The least square method [Alligier et al., 2012]

4 Results

A Point Mass Model



$$m.\,dV\ s = Thr - 0 - m.g.\sin(!)$$

A simplified model (longitudinal+vertical)

$$\underbrace{V_{TAS}.\frac{dV_{TAS}}{dt} + g.\frac{dz}{dt}}_{\text{energy-rate}} = \underbrace{\frac{(Thr - D).V_{TAS}}{m}}_{\frac{\text{power}}{\text{mass}}}$$

- z: altitude
- Thr (Thrust): thrust of the engines
- D (Drag): drag of the aircraft
- m: mass
- V_{TAS} (True Air Speed): velocity in the air
- $\frac{dV_{TAS}}{dt}$: longitudinal acceleration
- $\frac{dz}{dt} = V_{TAS}.sin(\gamma)$: rate of climb

The BADA contribution

$$\underbrace{V_{TAS}.\frac{dV_{TAS}}{dt} + g.\frac{dz}{dt}}_{\text{energy-rate}} = \underbrace{\frac{(Thr - D).V_{TAS}}{m}}_{\frac{\text{power}}{\text{mass}}}$$

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BADA model

- Max climb thrust:
  $Thr = f(T, V_{TAS}, z)$
- Drag:
  $D = f(T, V_{TAS}, z, m)$

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Equation at a given point

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Using radar and weather data, we know :

- T, V_{TAS}, z, $\frac{dz}{dt}$, $\frac{dV_{TAS}}{dt}$

We want to adjust the *mass m*

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$$\underbrace{E}_{\text{energy-rate}} = \underbrace{f(T, V_{TAS}, z, m)}_{\text{BADA model}}$$

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Using radar and weather data, we know :

- T, V_{TAS}, z, $\frac{dz}{dt}$, $\frac{dV_{TAS}}{dt}$

We want to adjust the *mass* m

$$\underbrace{E}_{\text{energy-rate}} = \underbrace{f(T, V_{TAS}, z, m)}_{\text{BADA model}} = \frac{P(m)}{m}$$

P, a known function

1. Computing the $\frac{power}{mass}$ provided by BADA
2. The adaptive method [Schultz et al., 2012]
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The adaptive method [Schultz et al., 2012]

Principle

- We assume an initial guess m_0
- At each point i, the mass m_i is estimated using m_{i-1}

$$E_i = \frac{P_i(m_i)}{m_i}$$

The adaptive method [Schultz et al., 2012]

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At each new point i, we have:

$$m_i = \frac{P_i(m_{i-1})}{E_i}$$

Introduction of the sensitivity parameter β [Schultz et al., 2012]

The previous update formula can be rewritten:

$$m_i = m_{i-1}\left[1 + \frac{m_{i-1}}{P_i(m_{i-1})}\underbrace{\left(E_i - \frac{P_i(m_{i-1})}{m_{i-1}}\right)}_{\substack{\text{error on the energy rate}\\ \text{when using } m_{i-1}}}\right]^{-1}$$

Introducing a sensitivity parameter β_i:

$$m_i = m_{i-1}\left[1 + \beta_i\frac{m_{i-1}}{P_i(m_{i-1})}\left(E_i - \frac{P_i(m_{i-1})}{m_{i-1}}\right)\right]^{-1}$$

Logic of the sensitivity parameter β [Schultz et al., 2012]

$$m_i = m_{i-1}\left(1 + \beta_i \frac{m_{i-1}}{P_i(m_{i-1})}\left(E_i - \frac{P_i(m_{i-1})}{m_{i-1}}\right)\right)^{-1}$$

Let $\Delta\dot{E}_i = \frac{1}{gV_{\text{TAS}}}\left(E_i - \frac{P_i(m_{i-1})}{m_{i-1}}\right)$, β is updated using this rule:

if $i > 0$ and $\Delta\dot{E}_i > 0.0001$

and $\frac{\Delta\dot{E}_i - \Delta\dot{E}_{avg}}{\Delta\dot{E}_{avg}} < 3$

then

$$\beta_i = max(0.205, \beta_{i-1} + 0.05)$$

else

$$\beta_i = 0.005$$

Logic of the sensitivity parameter β [Schultz et al., 2012]

$$m_i = m_{i-1}\left(1 + \beta_i \frac{m_{i-1}}{P_i(m_{i-1})}\left(E_i - \frac{P_i(m_{i-1})}{m_{i-1}}\right)\right)^{-1}$$

This mechanism increases robustness

- If $\Delta\dot{E}_i$ repeatedly high in the same order of magnitude, β will increase, strengthening adaptation
- Isolated low or high $\Delta\dot{E}_i$ has a lower impact on adaptation

The variation is limited

- The variation is limited to 2% of the reference mass
- The estimated mass is kept within 80% and 120% of the reference mass

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The least square method [Alligier et al., 2012]

Principle

- All the points are considered at once
- Minimizes the sum of square error on the energy rate

At each point i, we have:

$$\frac{P_i(m_i)}{m_i} = E_i$$

However, the different masses are not independant variables
⇒ These equations above cannot be satisfied altogether (in general)
Then, we search $(m_1, \ldots, m_n)$ minimizing:

$$E(m_1, \ldots, m_n) = \sum_{i=1}^{n} \left(\frac{P_i(m_i)}{m_i} - E_i \right)^2$$

Relationship between the m_i

fuel consumption

BADA model of the fuel consumption:

$$\frac{dm}{dt} = -f(T, V_{\mathrm{TAS}}, z)$$

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$$m_i = m_n + \int_{t_i}^{t_n} f(T(t), V_{TAS}(t), z(t))\,dt$$

$$\Rightarrow m_i = m_n + \sum_{k=i}^{n-1} \frac{f(t_{k+1}) + f(t_k)}{2}(t_{k+1} - t_k)$$

$$\Rightarrow m_i = m_n + \delta_i$$

The error function can be rewritten:

$$E(m_1,\ldots,m_n) = E(m_n) = \sum_{i=1}^{n}\left(\frac{P_i(m_n+\delta_i)}{m_n+\delta_i} - E_i\right)^2$$

Minimizing this error

The error function can be rewritten:

$$E(m_1, \dots, m_n) = E(m_n) = \sum_{i=1}^{n} \left(\frac{P_i(m_n + \delta_i)}{m_n + \delta_i} - E_i \right)^2$$

Minimizing this error can be done by solving:

$$E'(m) = 0$$

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With the BADA model, P_i polynomial of the second degree

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Minimizing this error can be done by solving:

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With the BADA model, P_i polynomial of the second degree
$\Rightarrow$ Solving $E'(m) = 0$ leads to find roots of a polynomial of degree at most $3(n-1) + 4$

Minimizing this error

The original error function:

$$E(m_n) = \sum_{i=1}^{n} \left(\frac{P_i(m_n + \delta_i)}{m_n + \delta_i} - E_i \right)^2$$

$\Rightarrow$ Numerical issues solving $E'(m) = 0$

Minimizing this error

The original error function:

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$\Rightarrow$ Numerical issues solving $E'(m) = 0$

An approximated error function:

$$E_{approx}(m_n) = \sum_{i=1}^{n} \left(\frac{P_i(m_n + \delta_i)}{m_n + \delta_{avg}} - E_i \right)^2$$

$$\text{with: } \delta_{avg} = \frac{1}{n} \sum_{i=1}^{n} \delta_i$$

Minimizing this error

The original error function:

$$E(m_n) = \sum_{i=1}^{n} \left(\frac{P_i(m_n + \delta_i)}{m_n + \delta_i} - E_i \right)^2$$

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$$\text{with: } \delta_{avg} = \frac{1}{n} \sum_{i=1}^{n} \delta_i$$

$\Rightarrow$ Solving $E'_{approx}(m) = 0$ leads to find roots of a polynomial of degree 4

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4 Results

Synthesized aircraft trajectories

Each trajectory

- BADA 3.9
- Trajectories start at altitude 12,000 ft
- Each 12 seconds, we observe: $T, V_{TAS}, z, \frac{dz}{dt}, \frac{dV_{TAS}}{dt}$
- Trajectories of 4 minutes long (ie. 21 points)

Each set of trajectories

- Contains 1,000 trajectories of a given aircraft type
- Distribution of the parameters used to generate trajectories

parameter	distribution
CAS	$CAS_{ref} + uniform([-30; 30])$
Mach	$Mach_{ref} + uniform([-0.03; 0.03])$
ΔT	$uniform([-20; 20])$
mass	$mass_{ref} \times uniform([0.8; 1.2])$

Adding a Gaussian noise

Principle

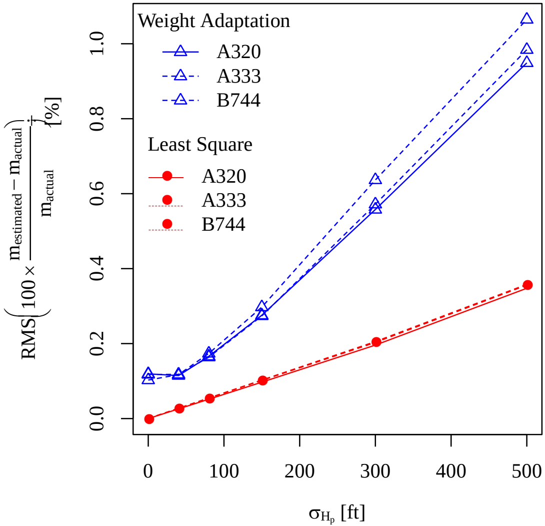
Given one variable among $T, V_{\text{TAS}}, z, \frac{dz}{dt}, \frac{dV_{\text{TAS}}}{dt}$ and a standard deviation σ:

1. We draw errors from the Gaussian distribution
2. For each observation of the choosen variable, the error is added to the observed value

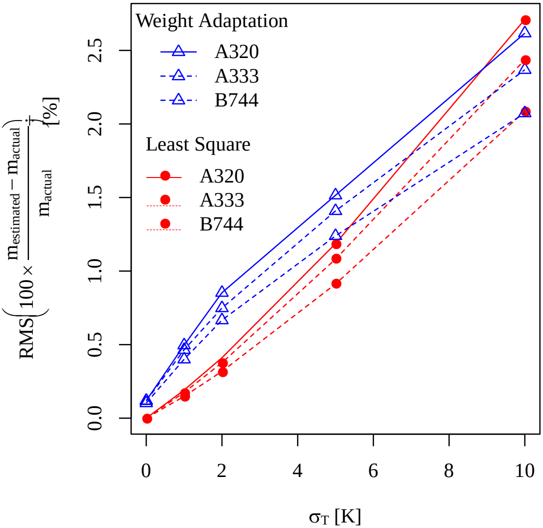
For instance, if we have chosen T and $\sigma_T = 2K$:

1. We draw $1,000 \times 21$ values from $N(0, 2)$
2. These $21,000$ values are added to the $21,000$ observations of T

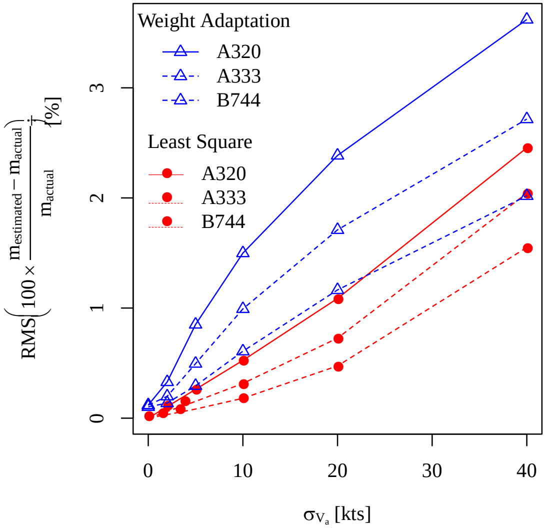
Results: Noise on z



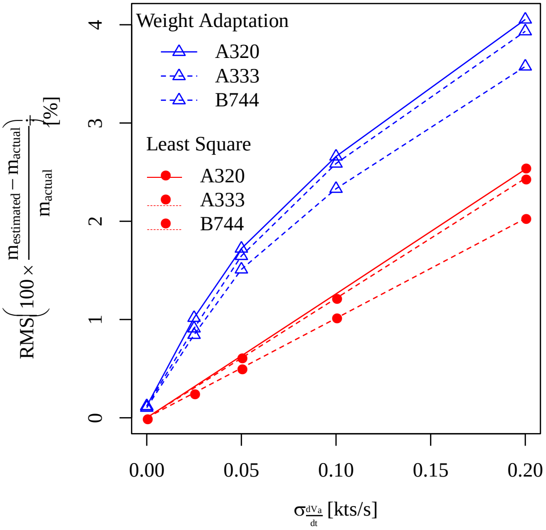
Results: Noise on T



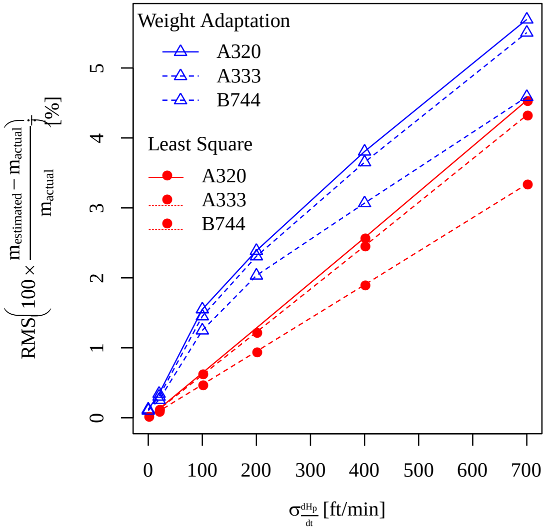
Results: Noise on V_{TAS}



Results: Noise on $\frac{dV_{TAS}}{dt}$



Results: Noise on $\frac{dz}{dt}$



Conclusion

accuracy

- Both methods gives a good estimation of the mass
- Least square method performs slightly better

Beyond accuracy

- Adaptive method is simpler to implement than the least square method
- Adaptive method can use a black box model of the power Adaptive
- method needs more points to give a good estimate of the mass
- Adaptive method demands to tune the β sensitivity parameter

Further work

- Comparing these two methods on real data
- Use the estimated mass in machine learning techniques

Thank you, any questions ?

Alligier, R., Gianazza, D., and Durand, N. (2012).
Energy Rate Prediction Using an Equivalent Thrust Setting Profile (regular paper).
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Schultz, C., Thipphavong, D., and Erzberger, H. (2012).
Adaptive trajectory prediction algorithm for climbing flights.
In *AIAA Guidance, Navigation, and Control (GNC) Conference*.