Airspace Sectorisation using Constraint-Based Local Search

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Executive Summary

- We implemented a prototype airspace sectorisation system using Constraint-Based Local Search (CBLS).
- CBLS is declarative framework for combinatorial optimisation that separates model and search.

Sectorisation

- Given a set of flights over some time horizon divide the airspace in sectors such that their workload is balanced.
- Many definitions of workload exist, but our model is workload definition agnostic.

Approaches

Two major approaches:

- Graph based: Make a graph based on flight routes. Nodes represent route crossings. Sectorisation is then a partition of nodes.
- Cell Based: Divide the airspace into cells. Then do some geometric reasoning to get the sectorisation.

We used a cell based approach.

Sectorisation from the Perspective of a Computer Scientist

- Sectorisation is an NP-complete problem.
- This means that there is no known algorithm that always efficiently solves the problem. If there was, then many unsolved problems in computer science would be answered.
- Most approaches try to use heuristics that work well in practice.
- By using a high-level declarative framework, we avoid having to write ad hoc algorithms.

Constraint Programming

Constraint Programming (CP) offers methods and tools for:

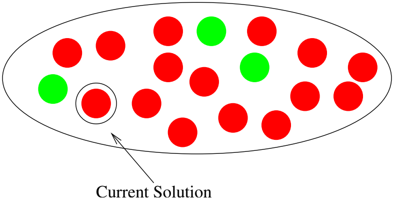
- Effectively modelling constraint problems.
- Efficiently solving constraint problems,
  by global search (in Sudoku fashion)
  or by local search.

Slogan of CP:

Constraint Program = Model + Search

Local Search

- Values are given to all the variables at the same time.
- Search proceeds by moves, which make small updates to complete assignments, upon probing the impacts of many candidate moves, called the neighbourhood.
- Stop when a good enough assignment has been found or when an allocated resource (running time, or a number of iterations) has been exhausted.



Constraint Based Local Search

- A problem is modelled as a conjunction of constraints, which declaratively encapsulate inference algorithms specific to common combinatorial substructures and are thus reusable.
- Each constraint has an associated penalty that measures how far the current solution is from satisfying the constraint.
- A master search algorithm operates on the model, guided by user-indicated/designed (meta-)heuristics. The main idea is to find moves that reduce the penalty.

Violations

Example ($\mathrm{AllDifferent}(\{x_1, x_2, x_3, x_4\})$)

- When $x_1 = 5$, $x_2 = 5$, $x_3 = 5$, and $x_4 = 6$:
  - The constraint violation is 2, since at least two variables must be changed to reach a satisfying assignment.
  - The variable violations of x_1, x_2, and x_3 are 1.
  - The variable violation of x_4 is 0.

This would guide the CBLS system to consider changing x_1,x_2 or x_3.

Constraint Programming = Model + Search

Model:

- Variables: One variable per cell as output from ASTAAC.
- Constraints
  - Balanced workload (soft) (Combination of conflict and monitoring workload)
  - Minimum dwell time (soft)
  - Convexity (soft) Trajectory based
  - Connectedness (hard)
  - Compactness (soft)
- The cost is the sum of the penalties of the constraints above and is minimised.

Constraint Programming = Model + Search

Search:

- Use standard local search techniques: tabu lists, random restarts ....
- Number of entry points is not minimised, because we conjecture that it is minimised as an effect of minimising the cost.
- Connectedness is our only hard constraint. Our initial solution is constructed to be connected. We then pick our neighbourhoods so that all moves preserve connectedness. This is a common technique in local search.

Convexity and Compactness

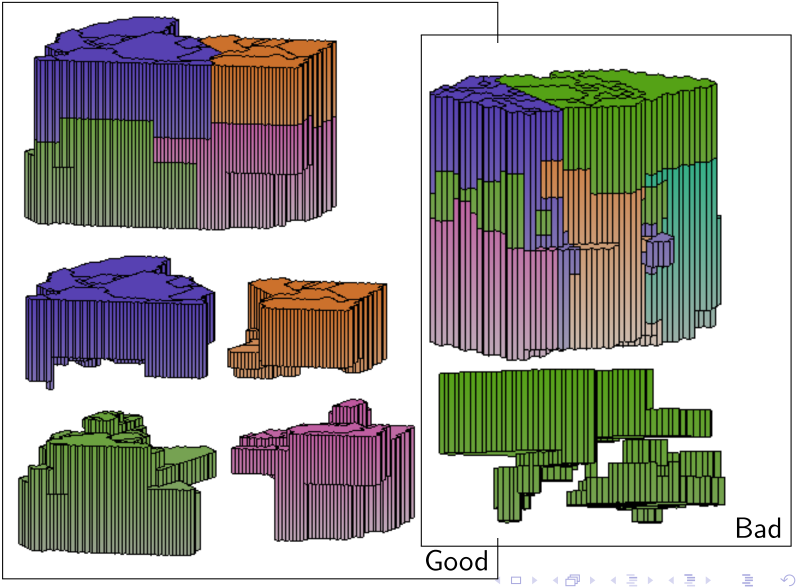
- Define convexity in terms of flight trajectories.
- Consider the sequence

$$[1, 1, 2, 1, 1, 1, 4, 4, 1, 1, 4, 4]$$

- Each number is the currently assigned cell.
- We penalise the number of reentries. Sector one is revisited twice and hence given a convexity penalty of two.

For compactness we minimise the surface area of each sector.

Results



Conclusions and Future work

- Using a declarative approach (CBLS) allowed easy prototyping of various alternatives.
- Future work includes looking more closely at compactness and develop more reusable constraints.
- Incorporating more realistic operational constraints.
- Dynamic baseline sectorisation.