

Regression Analysis of Top of Descent Location for Idle-thrust Descents

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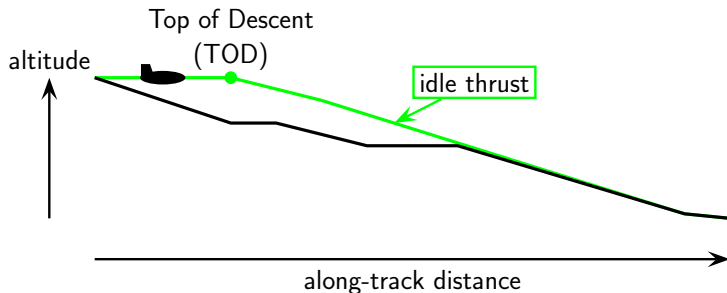
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Vertical profile of aircraft descent



Idle-thrust descent reduces fuel consumption and emissions.

Why is predicting vertical profile difficult?

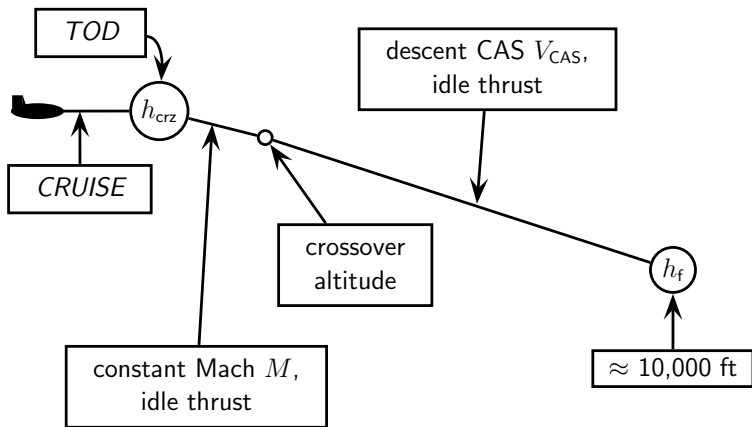
- Missing information?
- Complicated structure of function?

Why is predicting vertical profile important?

- Avoid conflicts with crossing traffic
- Improve along-track predictions

Statistical approach to understand nature of problem

Descent Constraints

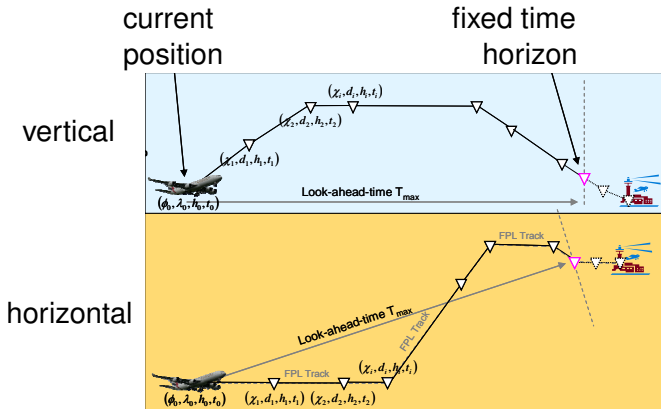


Descent Procedure

- Boeing 737-800 aircraft equipped with ADS-C (ground station initiates contract with aircraft)
- Crew loaded Standard Terminal Arrival Route (STAR) into FMS about 45 min prior to arrival
- Controller sometimes verbally changed speed profile
- FMS computed idle-thrust descent

Downlink of FMS Intent Path

FMS intent path in Intermediate Projected Intent (IPI) part of ADS-C messages



Horizontal descent length S_{TOD} between TOD and path end as a function of

- cruise altitude h_{crz}
- constant Mach segment speed M
- constant CAS segment speed V_{CAS}
- final altitude h_f
- integral W of along-track wind

Horizontal descent length S_{TOD} from IPI
as a function of

- cruise altitude h_{crz} from IPI
- constant Mach segment speed M
- constant CAS segment speed V_{CAS}
- final altitude h_f from IPI
- integral W of along-track wind

Horizontal descent length S_{TOD} between TOD and path end as a function of

- cruise altitude h_{crz}
- constant Mach segment speed M from ADS-C report prior to TOD
- constant CAS segment speed V_{CAS} estimated based on ADS-C state data
- final altitude h_f
- integral W of along-track wind

- Wind forecast used by FMS not available for this analysis
- Estimated based on World Area Forecast System
- For consistency with FMS:
 - Used wind profile at closest grid point to destination airport
 - Used winds at 10,000 ft, FL180, FL300 and cruise level
- Contribution of wind to S_{TOD} is

$$W = \int_{t_1}^{t_2} \hat{w} dt,$$

where \hat{w} is approximately tailwind.

Overview of Descents Analyzed

- 1088 Boeing 737-800 descents flown by Qantas February 2009 – September 2011 into Melbourne, Australia
- “Typical” descent
 - h_{crz} is FL380–390
 - $h_f \approx 10,400$ ft
 - $V_{CAS} \approx 280$ KCAS

but values seem sufficiently spread out for regression analysis

- 11% had CFM56-7B26 engines; others had CFM56-7B24 engines

$$S_{\text{TOD}} = \beta_0 + \beta_1 h_{\text{crz}} + \beta_2 h_{\text{f}} + \beta_3 M + \beta_4 V_{\text{CAS}} + \beta_5 W + \epsilon$$

- Could include higher-order terms such as W^2 or $h_{\text{f}} V_{\text{CAS}}$
- Overdetermined system, so

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} \|\epsilon\|^2$$

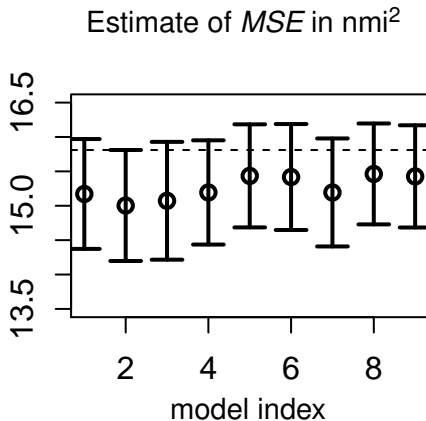
- Adding more terms will decrease residuals but may result in poor prediction

- Hypothesis testing with significance level of 0.05, say
 - incorrectly accept a term 5% of time
 - incorrectly reject a term unknown percentage of time
- K -fold cross-validation
 - Partition samples into K sets
 - Do K fits, each using one subset for testing and the rest for training
 - Gives estimate of mean square error MSE

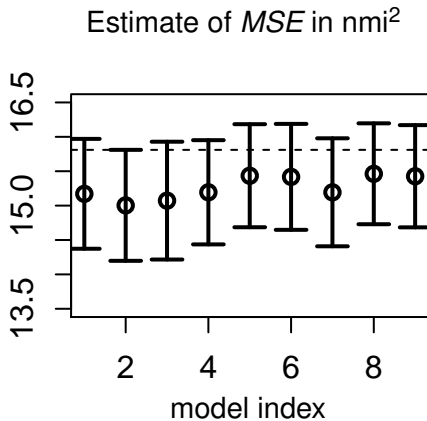
First-order Models Analyzed

index	terms included
1	$h_{crz}, h_f, V_{CAS}, M, W$ with different coefficients for the two different engine types
2	$h_{crz}, h_f, V_{CAS}, M, W$
3	h_{crz}, h_f, V_{CAS}, M
4	h_{crz}, h_f, V_{CAS}, W
5	$\Delta h, V_{CAS}, M, W$
6	$\Delta h, V_{CAS}, M$
7	h_{crz}, h_f, V_{CAS}
8	$\Delta h, V_{CAS}, W$
9	$\Delta h, V_{CAS}$

10-fold Cross-validation for First-order Models



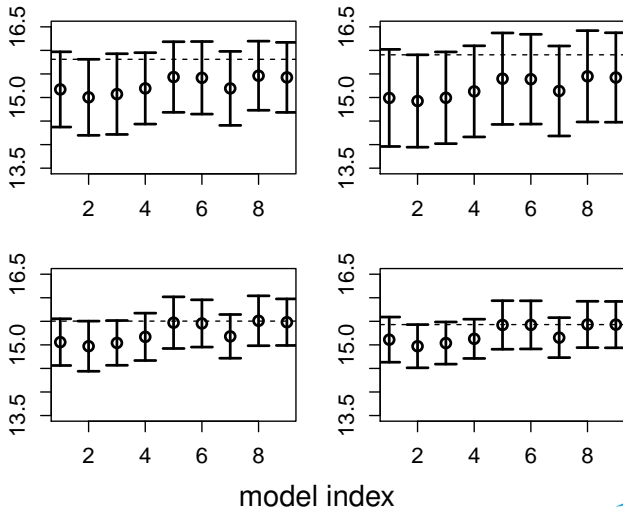
10-fold Cross-validation for First-order Models



Hypothesis testing: model 9 worse than 7 (separate altitudes)
worse than 3 (Mach number)

10-fold Cross-validation for First-order Models

Estimate of MSE in nmi^2

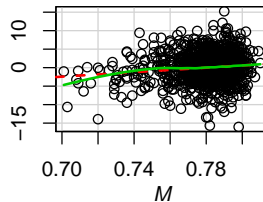
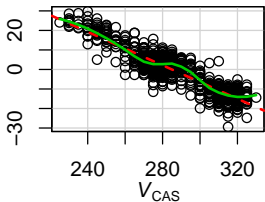
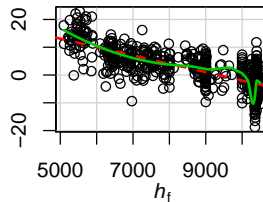
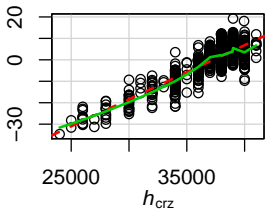


Partial Residual Plots

Relationship between S_{TOD} and one independent variable given the other predictors

smooth fit of points

least squares fit



- Model 3: $S_{\text{TOD}} \approx \beta_0 + \beta_1 h_{\text{crz}} + \beta_2 h_f + \beta_3 V_{\text{CAS}} + \beta_4 M + W$
large $\text{var}(\hat{\beta}_4)$ because observed M not spread out enough
- Model 7: $S_{\text{TOD}} \approx \beta_0 + \beta_1 h_{\text{crz}} + \beta_2 h_f + \beta_3 V_{\text{CAS}} + W$
- Model 9: $S_{\text{TOD}} \approx \beta_0 + \beta_1 (h_{\text{crz}} - h_f) + \beta_3 V_{\text{CAS}} + W$
- Goodness of fit
 - Estimate of standard deviation of error is 3.9 nmi
 - Fit is within 5 nmi of observed S_{TOD} for 82% of samples
- For comparison, standard deviation of S_{TOD} is 12.4 nmi

- Approximations such as computation of W
- Missing data
- Unavailability of aircraft mass especially noteworthy
 - Would need at least 1000 samples to test this
 - Not likely to be available for ground automation

Regression analysis of the FMS-computed idle-thrust descent TOD location for over 1000 descents in Qantas B737-800 aircraft to Melbourne, Australia fit to

$$S_{\text{TOD}} \approx \beta_0 + \beta_1 h_{\text{crz}} + \beta_2 h_f + \beta_3 V_{\text{CAS}} + W$$

- Estimate of standard deviation of error is 3.9 nmi.
- Over 80% of the residuals are less than 5 nmi in absolute value.
- All diagnostics give good results.
- More accurate than demonstrated for other ground automation predictors

- Can similar methods be used to better understand causes of errors for other ground automation predictors?
- Can regression models be developed for other aircraft types, locations, airlines, and such?
- Can regression models be used in ground automation?
 - Fast predictions of TOD for “what-if” capability to help controller choose descent speed
 - Use when accurate aircraft performance models unavailable — but need sufficient historical data