Statistical prediction of aircraft trajectory: regression methods vs point-mass model

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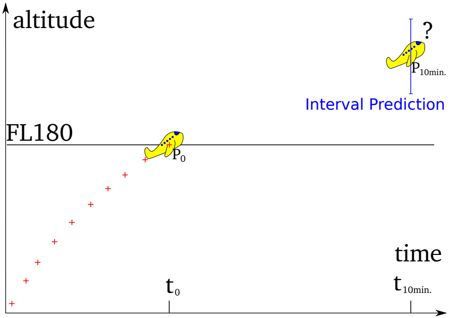
Introduction

Predict an interval on future altitude :



Introduction

Motivations

- **Ground-based** prediction of the altitude during climb
- Conflict detection and resolution purposes
  - Time horizon : 10-20 minutes
  - Need to evaluate uncertainty

Two approaches

- BADA model
- Regression models

The recorded data (july 2006, January 2007)

- Radar plots of 14 941 A320 from Paris Orly or Roissy
  1 *plot every 1 to 3 secs.*
- Weather data from the ALADIN model (Météo France)
- Variables calculated
  - Relative wind (W_{along}, W_{cross})
  - Distance flown
  - Curvature
  - Bank angle
  - $(\Delta T_0)_{\text{ISA}}$
  - CAS (calibrated air speed)
  - M Mach number
  - ... and their variations

Two methods to make predictions

Using a point-mass model

- Most common way
- Requires knowledge of many parameters
  mass, target speeds, thrust law, aircraft operation, etc.

Using a regression method :

$$\textit{future position} = f(\textit{known variables}\,;\textit{parameters})$$

- Which variables ?
  current state, past positions, wind, temperature, etc.
- Which function f ?
  linear, polynomial, neural network, KNN, etc.

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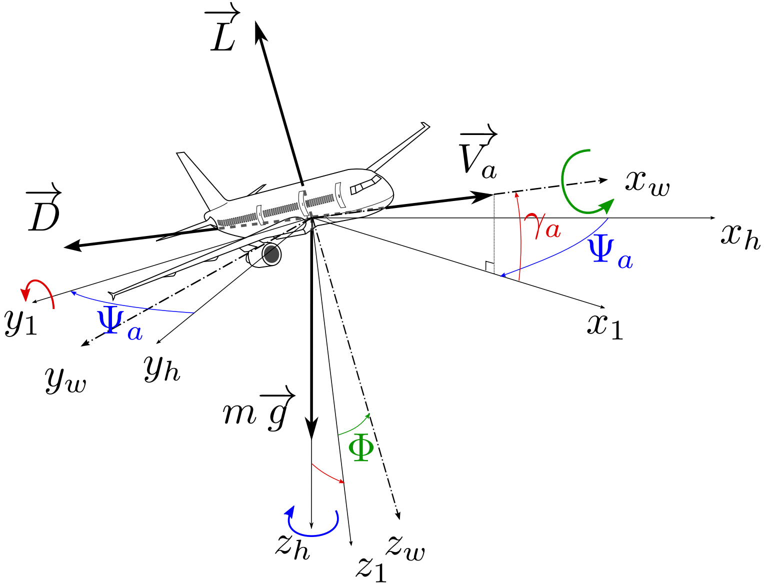
The point-mass model



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Total energy variations

Longitudinal acceleration $\frac{dV_a}{dt}$ and ROCD $\frac{dH_p}{dt}$:

$$\boxed{\underbrace{\frac{Power}{m}}_{\text{specific power}} = \underbrace{V_a \frac{dV_a}{dt} + g_0 \left(\frac{T}{T - \Delta T} \right) \frac{dH_p}{dt}}_{\text{specific energy rate}} + \underbrace{\frac{d\overrightarrow{W}}{dt}}_{\text{wind effect}} . \overrightarrow{V_a}} \quad (1)$$

$$Power = (Thr - D) \times V_a \quad (2)$$

The thrust (Thr) and drag (D) forces are given by the BADA model.

Energy share : acceleration or climb ?

Energy share factor :

$$ESF = \frac{g_0 . \left(\frac{T}{T - \Delta T}\right) . \frac{dHp}{dt}}{V_a \frac{dV_a}{dt} + g_0 \left(\frac{T}{T - \Delta T}\right) \frac{dHp}{dt}} = \left[1 + \left(\frac{T - \Delta T}{T}\right) \left(\frac{V_a}{g_0}\right) \left(\frac{dV_a}{dH_p}\right) \right]^{-1} \tag{3}$$

Prediction with point-mass models

What is required ?

- Total energy model equation + forces model
- ESF law *CAS/Mach climb ? constrained ROCD ?*
- Model parameters *Mass ? thrust law (max climb, reduced) ?*
- Meteo *Wind ? temperature ?*

A lot of unknowns values, and uncertainties.

Basic assumptions (BADA model) :

- standard CAS/Mach climb
- reduced power climb
- reference mass

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Statistical model : $y = f(x, \theta) + \epsilon$

- y variable to predict
  altitude, distance
- x vector of p explanatory variables
  past positions, air speed, Mach number, rate of climb, wind, temperature, etc.
- f a function to choose
  linear, polynomial, neural network, etc
- θ parameters to adjust according observed instances
- ϵ : residual.

Adjusting the parameters θ

Observed values of (x, y) :

$$\begin{array}{c} \begin{array}{cc} \overbrace{\hphantom{x_1^{(1)} \quad \dots \quad x_p^{(1)}}}^{x} & \overbrace{\hphantom{y^{(1)}}}^{y} \end{array} \\ \begin{pmatrix} x_1^{(1)} & \dots & x_p^{(1)} & y^{(1)} \\ \vdots & & \vdots & \vdots \\ x_1^{(n)} & \dots & x_p^{(n)} & y^{(n)} \\ \vdots & & \vdots & \vdots \\ x_1^{(N)} & \dots & x_p^{(N)} & y^{(N)} \end{pmatrix} \end{array}$$

Choose θ so as to minimize the expectation of loss L :

$$\hat{\theta} = argmin\ E\Big(L(Y, f(x, \theta))\Big)$$

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Interval prediction for a new instance x

$Y = F(x) + \varepsilon$

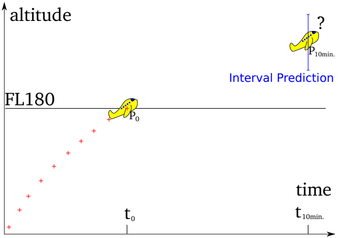
- ε is normal, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$
- quadratic loss function : $L(y, f(x,\theta)) = (y - f(x,\theta))^2$
- $\hat{\theta} = \underset{\theta \in \Theta}{\arg\min} \left[\sum_{n=1}^{N} (y_n - f(x_n, \theta))^2 \right]$
  sometimes weighted : $\min \sum_{n=1}^{N} \beta_n (y_n - f(x_n,\theta))^2$
- $f(x, \hat{\theta})$ is the estimation of $F(x)$, the "true" model
  $E(f(x,\hat{\theta})) = F(x), \quad f(x,\hat{\theta}) \sim \mathcal{N}(F(x), \sigma^2_{pred})$

Confidence intervals $P(Y(x) \in I(x)_{1-\alpha}) = 1 - \alpha$

- Training set : $RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (y_n - f(x_n, \hat{\theta}))^2}$
- Interval : $I(x)_{1-\alpha} = f(x, \hat{\theta}) \pm Z_{\frac{\alpha}{2}} * RMSE$

Interval prediction for climbing aircraft

- Calibrate a model, using recorded trajectories
- For a new aircraft trajectory, we have “in theory” :
  $P(\textit{aircraft} \in \textit{its predicted interval}) \geq 1 - \alpha$.



Actual ratio of aircraft actually within confidence interval ?

Regression methods

Three methods were tested :

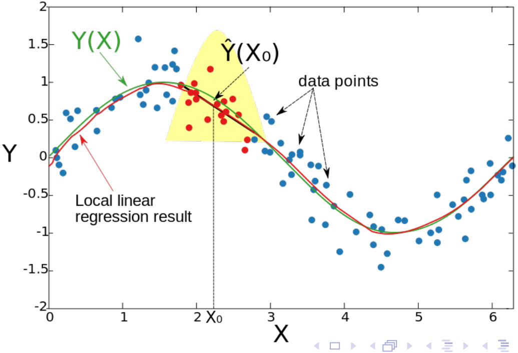
- Ordinary linear least squares

$$f(x, \theta) = \theta_0 + \sum_{i=1}^{p} \theta_i x_i \qquad , j \in [1 \ldots q]$$

- Locally weighted linear model
  $\rightarrow$ LOESS
- Non-linear least squares regression
  $\rightarrow$ Neural networks

Local linear model [Stone, 1977], [Cleveland, 1979]

- Assumes the model is locally linear.
- Minimizes least squares weighted by a kernel function (distance function)
  $\hat{\theta}_x = \arg\min \sum_{n=1}^{N} K_b(x - x_n)(y_n - x_n^T \theta_x)^2$



Local linear model

Some theoretical results

- [Fan, 1992], [Fan, 1993] show that local linear regression has a high asymptotic efficiency among all possible linear smoothers
- Other theoretical results on local linear regression by [Fan, 1993], [Fan and Gijbels, 1992], [Hastie and Loader, 1993], etc.

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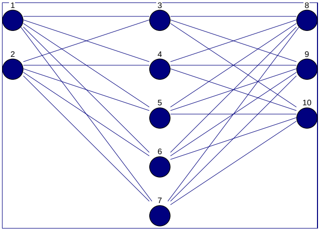
Neural network



$$f(x,\theta) = \Psi\Big(\sum_{j=1}^{h} \theta_{kj}\Phi\Big(\sum_{i=1}^{p} \theta_{ji}x_i + \theta_{j0}\Big) + \theta_{k0}\Big) \qquad (4)$$

- $\Phi(s_j) = \frac{1}{1+e^{-s_j}}$
- $\Psi(s_k) = s_k$

Experimental setup

Dimensionality reduction

Principal Component Analysis

(Karhunen-Loève transform)

76 variables $\rightarrow$ 15 principal components

Altitude and interval prediction with :

- **BADA** : ref. mass, standard CAS/Mach climb at reduced power thrust
- **BADA(obs)** : the same, but with observed CAS
- **LR** : linear regression
- **LOESS** : locally weighted linear model
- **NN** : neural network

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Results on altitude prediction

Method	MAE	RMSE
BADA	1440 (79)	1824 (95)
BADA(obs)	1440 (77)	1819 (86)
LR	744 (55)	962 (72)
NN	841 (47)	1080 (55)
Loess	699 (54)	908 (72)

TABLE: Average prediction errors, using 15 principal components.

Results on interval prediction

Method	Ratio in theoretical 95% interval	Theoretical 95% $\|\delta z\|$
BADA	0.92 (0.025)	3279
BADA(obs)	0.93 (0.021)	3369
LR	0.94 (0.013)	1863
NN	0.905 (0.030)	1846
Loess	0.948 (0.022)	1842

TABLE: Uncertainty on the altitude prediction (Airbus A320), for a reference point at FL180 and a 10-minutes look-ahead time.

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Conclusion

- Regression significantly better than the BADA model
- Local linear models, most efficient in point and interval estimation

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Further work and study

- Different regressions like Gaussian Process, Quantile Regression
- Other intervals like tolerance interval
- Time series model
- Several aircraft types

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Thank you & questions ?

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