

# Model for Longitudinal Uncertainty during Controlled Time of Arrival Operations

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**Abstract**—This paper presents a model to estimate the longitudinal uncertainty of an aircraft’s future trajectory while flying towards a Controlled Time of Arrival (CTA). Uncertainties during such operations pose challenges to Air Traffic Control (ATC) and are mostly caused by meteorological forecast errors driving the associated speed corrections applied by the guidance system to meet the CTA. Firstly, the model in this paper can be used to estimate the probability of spacing reductions between two in-trail aircraft performing a CTA operation. Secondly, the model allows for predicting the upper and lower bound of the possible speed corrections to meet a CTA. Thirdly, the model can be used to predict the effect of meteorological uncertainty on the range of achievable times an aircraft can reliably meet at the CTA fix. Finally, as this range of achievable times depends on the time or distance to go to the CTA fix, the model can be used to assess when or where this window will be maximal which is relevant to arrival management systems. For a popular range of aircraft types and flight conditions, 1 hour was found to be an appropriate average horizon for CTA allocation. The model was applied to a recorded set of arrival track data from Melbourne airport upon which several operational considerations were made with respect to the anticipated use of CTA.

**Keywords**—Controlled Time of Arrival; Trajectory Based Operations; Trajectory Uncertainty; 4D-Trajectory; Arrival Management

## I. INTRODUCTION

Future concepts for Air Traffic Management (ATM) envision increased delegation of responsibilities to advanced airborne automation. An example thereof is the use of airborne Required Time of Arrival (RTA) functionality, a feature of a modern Flight Management System (FMS) designed to calculate and adjust the speed of the aircraft in an attempt to arrive at a point in space within a tolerance of a defined target time, also referred to as Controlled Time of Arrival (CTA). CTA forms the cornerstone of the SESAR ATM Master Plan Step 1, Time-based Operations (Initial 4D) [1]. In parallel, FAA’s NextGen Implementation Plan 2012 states that “Enhancements to the navigation capabilities of aircraft, RNAV/RNP with Time of Arrival Control (TOAC) in the descent phase, will begin to increase benefits of trajectory operations through the adaptability of the aircraft trajectory to enable operational predictability and arrival accuracy of aircraft” [2].

While many trials around the world have demonstrated that some modern aircraft are capable of performing a CTA operation to very tight tolerances [3][4], research suggests that additional work is required to mature the concept [5][6][7]. The trial Initial 4D flight of February 2012 operated by Airbus in cooperation with the Maastricht Upper Area Control Centre (MUAC) and the North European and Austrian Consortium (NORACON) [8] demonstrated the aircraft down-linking its trajectory and having the ground system coordinate a required time for it to cross at a waypoint. The flight was a successful demonstration of the Initial 4D technical capability, however it raised some issues to be addressed, including uncertainty to Air Traffic Control (ATC) of the magnitude and timing in which the aircraft is going to change its speed as it attempts to achieve the time constraint. De Smedt and Klooster (2011) [9] investigated the probability of a trailing aircraft being able to meet a time constraint either 90s or 120s behind the time constraint of a leading aircraft, as function of the initial conditions of both aircraft. In addition the paper investigated the probability that either the actual spacing or the predicted spacing in three minutes look-ahead time between the two aircraft, would reduce below the separation minima. Although the results were technically on the optimistic side, with time-constraint-achievability rates of around 82% and separation losses below 5%, it was recommended that additional ATC support tools are required. In a different study De Smedt, Bronsvort and McDonald (2013) [7] used an actual arrival sequence at Melbourne, Australia to investigate if the concept of CTA could result in more efficient arrival trajectories. This study concluded that in high density traffic situations, the ability to absorb delay through only the use of CTA is not sufficient, and requires sequence resolutions to be generated external to the FMS, e.g. path stretches or level changes. This paper aims to address the before mentioned issues by presenting a generic model to estimate longitudinal uncertainty during a typical CTA operation.

This paper is organized as follows: first, a mathematical model will be derived which predicts the longitudinal uncertainty of an aircraft’s predicted trajectory and the magnitude of required groundspeed correction to achieve a CTA in the presence of an assumed wind error. Then this model is used to predict a range of achievable times which an aircraft can meet reliably at a CTA fix in the presence of this

wind error. Obviously this range of achievable times is smaller than the range of achievable times an aircraft would be able to meet with perfect meteorological forecasts. Analysis of the maximum and minimum (or zero) of this range of achievable times will be performed. Finally the theory will be applied to the recorded set of arrival track data from Melbourne airport.

## II. TRAJECTORY UNCERTAINTY MODEL FOR OPERATIONS USING CONTROLLED TIME OF ARRIVAL

When a CTA, issued by ATC, is inserted as a constraint in an aircraft's FMS, this system will compute a trajectory with a speed profile generating an arrival time at the CTA fix which equals the CTA. If during the flight, the trajectory is affected by unexpected disturbances (for example wind changes) the FMS will make a correction to the speed profile so that the aircraft still arrives on time within the tolerance. This means that the initial condition of the aircraft at the time when the CTA is issued and the condition of the aircraft at the CTA fix are more or less known (bounded by preconditions). The condition of the aircraft in the middle of the trajectory, between the starting point and the CTA fix, is uncertain due to the initially unknown perturbations and required speed corrections. When the uncertainty around an aircraft's trajectory during a CTA operation would be plotted, it would have a profile which is "clamped" at the start and end points [10]. Next, a mathematical model will be proposed to compute this uncertainty, expressed as a longitudinal along-track position (or distance) error as a function of time.

Assume  $x_{pred}$  the longitudinal along-track position of an aircraft's predicted horizontal trajectory and  $x_{act}$  the longitudinal along-track position of the aircraft's actually flown horizontal trajectory.  $x_{pred}$  can be expressed in function of the predicted groundspeed profile  $v_{pred}(t)$  of the aircraft and time  $t$  as follows:

$$x_{pred} = \int v_{pred}(t) \cdot dt \quad (1)$$

Assuming that the actual trajectory which the aircraft flies is affected by a wind uncertainty  $w(t)$ , a speed correction  $s(t)$  will need to be applied if the original arrival time of the predicted trajectory needs to be maintained. Mathematically,  $x_{act}$  can be written in function of  $v_{pred}(t)$ ,  $w(t)$  and  $s(t)$  as follows:

$$x_{act} = \int [v_{pred}(t) + w(t) - s(t)] \cdot dt \quad (2)$$

The difference between  $x_{act}$  and  $x_{pred}$  can be defined as the longitudinal position uncertainty  $x$  of the predicted trajectory:

$$x = x_{act} - x_{pred} \quad (3)$$

After substituting  $x_{act}$ ,  $x_{pred}$  and differentiating the equation, this yields:

$$\frac{dx}{dt} = w(t) - s(t) \quad (4)$$

The above equation indicates that the along-track position uncertainty of a trajectory bounded by a CTA constraint is independent from the predicted groundspeed profile of that

trajectory. In the absence of other disturbances, it only depends on the wind uncertainty and the speed correction function. Theoretically, this is only true assuming that the vertical profile is constant and not affected by the speed corrections.

Assume that the wind uncertainty  $w(t)$  is constant in time. Just after the start of the CTA operation at time  $t_0$ , before any speed correction is made, the position uncertainty  $x$  will be linearly increasing as indicated in Figure 1.

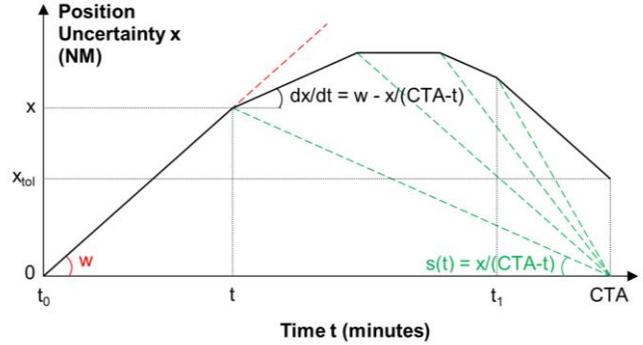


Figure 1. Position uncertainty versus time during CTA operation.

After an elapsed time  $t$ , the system will apply a speed correction  $s(t)$ . Assuming that the system does not know that there will be another wind error ahead (there is no wind blending of measured winds in forecast winds),  $s(t)$  will be such that in the absence of further wind error, the longitudinal position error will be zero again at the CTA time. This allows us to express  $s(t)$  mathematically as follows:

$$s(t) = \frac{x}{CTA - t} \quad (5)$$

Should the wind error continue after the speed correction, then the resulting longitudinal error is the sum of the wind error and the speed correction applied so far, integrated over time. At a certain time  $t_1$  the rate of change of the position uncertainty over time becomes such that in the absence of further speed corrections, the remaining position error at the CTA fix will be equal to a target tolerance  $x_{tol}$ . In other words, beyond time  $t_1$ , no further speed corrections are necessary to arrive on time at the CTA fix within the tolerance  $x_{tol}$ . Substitution of (5) in (4) yields the following differential equation for any time  $t \leq t_1$ :

$$\left. \frac{dx}{dt} \right|_{t \leq t_1} = w - \frac{x}{CTA - t} \quad (6)$$

Beyond  $t_1$ , the rate of change of the position error over time is constant and can be expressed as:

$$\left. \frac{dx}{dt} \right|_{t \geq t_1} = \frac{x - x(t_1)}{t - t_1} = \frac{x_{tol} - x(t_1)}{CTA - t_1} = w - \frac{x(t_1)}{CTA - t_1} \quad (7)$$

Assuming a constant wind error  $w$ , equation (6) becomes a non-homogeneous differential equation of the first order which can be solved using the method of integrating factors. Solving the differential equation yields the following expression for the longitudinal position uncertainty  $x$ :

$$x(t)_{t \leq t_1} = w \cdot (t - \text{CTA}) \cdot \ln\left(1 - \frac{t - t_0}{\text{CTA} - t_0}\right) \quad (8)$$

Solving equation (7) allows computing  $t_1$ :

$$t_1 = \text{CTA} - \frac{x_{\text{tol}}}{w} \quad (9)$$

By substituting equation (9) in equation (8),  $x(t_1)$  can be calculated after which  $x(t_1)$  and  $t_1$  can be substituted further in equation (7) to compute the position uncertainty after time  $t_1$ :

$$x(t)_{t \geq t_1} = w \cdot (t - \text{CTA}) \cdot \left(1 - \ln\left(\frac{w \cdot (\text{CTA} - t_0)}{x_{\text{tol}}}\right)\right) + x_{\text{tol}} \quad (10)$$

Finally, as it was assumed that after time  $t_1$ , no further speed correction was necessary to arrive within the tolerance  $x_{\text{tol}}$  at the CTA fix, the total amount of speed correction  $s$  that needed to be applied can be calculated as follows:

$$s = s(t_1) = \frac{x(t_1)}{\text{CTA} - t_1} = w \cdot \ln\left(\frac{w \cdot (\text{CTA} - t_0)}{x_{\text{tol}}}\right) \quad (11)$$

According to equation (11), the total magnitude of speed correction that is required to correct a trajectory so that it complies with a CTA, depends on the tolerance  $x_{\text{tol}}$ , the elapsed time ( $\text{CTA} - t_0$ ) to the CTA fix and the forecast error  $w$ . Note that this formula was derived assuming that  $w$  was constant over time. Therefore a maximum worst case value of  $w$  could be considered, for example 10kts. This equals the assumed groundspeed uncertainty for the means of compliance of the Time of Arrival Control function specified in [11]. Note that  $x_{\text{tol}}$  is expressed as a distance. Usually an aircraft's Time of Arrival Control System controls the arrival time to a defined time tolerance (for example 10 or 30 seconds). As it is expected that, especially for arriving aircraft in the terminal area, time constraints will be associated with an "AT or BELOW" speed constraint at the CTA fix, such time tolerances can easily be converted to an equivalent distance tolerance. For example, considering an arrival speed of 250kts in the terminal area, 10 seconds of time tolerance would correspond to approximately 0.7NM of distance tolerance. In cruise, assuming a nominal cruise speed of 450kts, 10 seconds of time tolerance would correspond to about 1.3NM of distance tolerance.

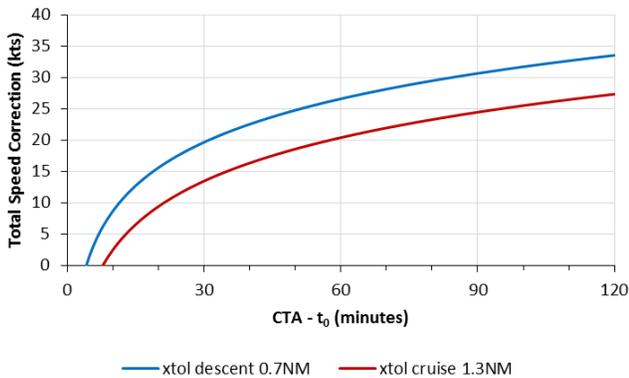


Figure 2. Total required speed correction in function of  $\text{CTA} - t_0$  and final tolerance  $x_{\text{tol}}$  assuming 10kts of wind error.

Figure 2 shows the output of formula (11) for the two tolerances and for an assumed maximum wind uncertainty of 10kts. Figure 2 indicates that for an elapsed time ( $\text{CTA} - t_0$ ) to the CTA fix of 30 respectively 90 minutes, 20 respectively 30 knots of total speed correction would be required to compensate a 10kts unforeseen wind error, considering a required tolerance of 0.7NM at the CTA fix. If the tolerance is relaxed to 1.3NM in cruise, the total speed correction reduces to 14 respectively 25kts. Figure 2 also indicates that the aircraft can fly a short amount of time without speed corrections and still remain within the required tolerance. For example if the tolerance is 0.7NM at the CTA fix, the aircraft can fly 4 minutes under 10kts of wind error without speed correction and still remain within the tolerance.

### III. VALIDATION OF THE TRAJECTORY UNCERTAINTY MODEL

Equations (8), (9) and (10) provide an easy, quick and analytical way to estimate the longitudinal position uncertainty around an aircraft's predicted trajectory at any time  $t$  between the time of allocation of the CTA (time  $t_0$ ) and the CTA. The model was validated against the output from a General Electric B737NG FMS workstation using software version U10.7. Three simulations were performed using the workstation in which the aircraft was descended from FL400 to 2000ft, flying to a CTA of 00:18:39 (starting from time 00:00:00). The wind error used in the simulations was 0 at FL400, then rising to respectively +15 and -15kts at FL390 after which it remained constant until 2000ft. The longitudinal position uncertainty of the original predicted profile assuming a 15kt forecast uncertainty was obtained by comparing the profiles flown with +/-15kts of wind error with the 0 wind profile. The outputs are presented by the red curves in Figure 3. The blue curves represent the output from the model derived above, assuming the same conditions (CTA 00:18:39 starting from time 00:00:00), wind error  $w$  +/-15kts and  $x_{\text{tol}}$  0.7NM). 15kts wind error was chosen instead of 10kts to ensure the FMS performed multiple speed corrections for illustrative purposes.

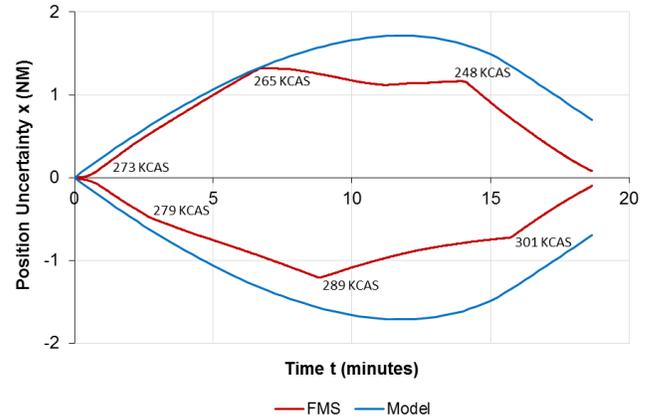


Figure 3. Comparison of trajectory uncertainty model with real FMS data assuming 15kts of wind error.

The difference in shape of the curves can be explained by the fact that firstly the forecast wind in the FMS is blended with the actual measured wind and therefore not constant, as illustrated in Figure 4. Secondly, in descent, the FMS adjusts

the Calibrated Airspeed (CAS) profile. Therefore the resulting increment in True Airspeed (TAS) varies with altitude. Finally, the design of the RTA function in the GE FMS is such that a new vertical profile is computed at each speed profile update.

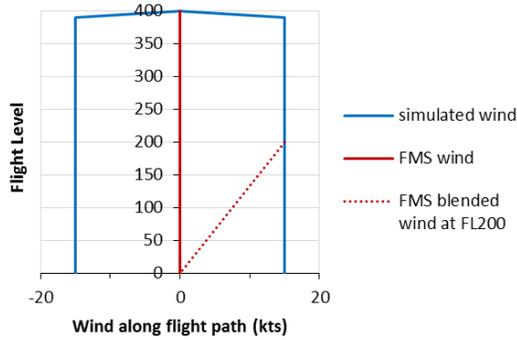


Figure 4. Winds in the simulation using the GE B737NG FMS workstation.

Still it can be observed that the output from the simulation is bounded conservatively by the output from the model. Figure 3 also indicates the CAS calculated by the FMS and flown by the aircraft during the simulation. Simulating a 15kts wind error, a total speed correction of respectively 28kts was required for the headwind case and 25kts for the tailwind case to arrive on time. Equation (11) from the derived model would yield a required speed correction of 28kts for the same conditions, which again is a very satisfactory result.

#### IV. COMPUTATION OF A RELIABLE $ETA_{MIN} - ETA_{MAX}$ WINDOW

From the previous paragraph it is obvious that the aircraft needs speed buffers to be able to compensate unknown forecast errors when flying to a CTA. These speed buffers need to be taken into account when computing a reliable earliest-latest time window in which a CTA can be achieved with a degree of certainty. This window is also referred to as reliable  $ETA_{min} - ETA_{max}$  window, in which  $ETA$  stands for Estimated Time of Arrival. Using the model presented above, in particular equation (11), such a reliable  $ETA_{min} - ETA_{max}$  can be estimated.

Let  $d$  be the distance to go,  $s$  the total speed correction required to compensate the forecast uncertainty and  $v_{min}$  and  $v_{max}$  the average minimum and maximum groundspeeds of the aircraft along the trajectory.  $ETA_{max}$  and  $ETA_{min}$  can be expressed as follows:

$$ETA_{max} = \frac{d}{v_{min}} \text{ and } ETA_{min} = \frac{d}{v_{max}} \quad (12)$$

The buffer to be subtracted from  $ETA_{max}$  to make  $ETA_{max}$  reliable in the presence of a forecast error  $w$ , can be calculated as follows:

$$\begin{aligned} \Delta ETA_{max} &= \frac{d}{v_{min}} - \frac{d}{v_{min} + s(ETA_{max})} \\ &= \frac{d \cdot s(ETA_{max})}{v_{min}(v_{min} + s(ETA_{max}))} \end{aligned} \quad (13)$$

As the second term containing  $s$  in the denominator of equation (13) is of lower order than the first term containing  $v_{min}$ , the relation can be rewritten as:

$$\frac{\Delta ETA_{max}}{ETA_{max}} \cong \frac{s(ETA_{max})}{v_{min}} = \frac{ETA_{max} \cdot s(ETA_{max})}{d} \quad (14)$$

A similar formula could be derived for  $\Delta ETA_{min}$ :

$$\frac{\Delta ETA_{min}}{ETA_{min}} \cong \frac{s(ETA_{min})}{v_{max}} = \frac{ETA_{min} \cdot s(ETA_{min})}{d} \quad (15)$$

In equations (14) and (15),  $s(ETA_{max})$  and  $s(ETA_{min})$  could be computed from equation (11) in which the CTA is replaced by respectively  $ETA_{max}$  and  $ETA_{min}$ . Thus the buffers applied to  $ETA_{max}$  and  $ETA_{min}$  to make the  $ETA_{min} - ETA_{max}$  window reliable can be computed from the  $ETA_{max}$  respectively  $ETA_{min}$ , the assumed maximum forecast error  $w$ , the distance to go  $d$  and the allowed tolerance at the CTA fix  $x_{tol}$ .

The reliable  $ETA_{min} - ETA_{max}$  window itself can then be expressed as:

$$\begin{aligned} |ETA_{min} - ETA_{max}|_{rel} &= ETA_{max} \left( 1 - \frac{\Delta ETA_{max}}{ETA_{max}} \right) \\ &- ETA_{min} \left( 1 + \frac{\Delta ETA_{min}}{ETA_{min}} \right) \\ &= ETA_{max} \left( 1 - \frac{ETA_{max} \cdot s(ETA_{max})}{d} \right) \\ &- ETA_{min} \left( 1 + \frac{ETA_{min} \cdot s(ETA_{min})}{d} \right) \end{aligned} \quad (16)$$

The model of equations (11), (14), (15) and (16) provides many options. If the  $ETA_{max}$  and  $ETA_{min}$  are known for a certain flight (through down-link from the aircraft's FMS or by computation using a ground Trajectory Predictor), the reliable window can easily be computed from equation (16) in which  $s$  represents the total speed correction which would be required to achieve a given CTA (in this case respectively the  $ETA_{max}$  and  $ETA_{min}$ ) and  $d$  represents the distance to go.

Equation (16) could also be rewritten as a function of the assumed average minimum and maximum groundspeeds  $v_{min}$  and  $v_{max}$ , taking into account equation (12):

$$\begin{aligned} |ETA_{min} - ETA_{max}|_{rel} &\cong \frac{d}{v_{min}} \left( 1 - \frac{s(ETA_{max})}{v_{min}} \right) \\ &- \frac{d}{v_{max}} \left( 1 + \frac{s(ETA_{min})}{v_{max}} \right) \end{aligned} \quad (17)$$

Figure 5 plots the output of equation (17), representing a reliable  $ETA_{min} - ETA_{max}$  window as a function of distance to go  $d$ . The solid blue and red curves represent the reliable window assuming nominal still wind conditions with a wind uncertainty  $w$  of 10 knots, a minimum cruise speed  $v_{min}$  of 410 KTAS, a maximum cruise speed  $v_{max}$  of 470 KTAS and a tolerance  $x_{tol}$  of respectively 1.3 and 3.9NM. With the assumed cruise speeds these tolerances correspond to a time tolerance of approximately 10, respectively 30 seconds. From Figure 5 it is clear that the  $ETA_{min} - ETA_{max}$  window reaches a maximum

value at a certain distance to go and thereafter decreases to zero. For a tolerance of 1.3NM in cruise, the distance to go at which the reliable window gets maximal in Figure 5 is about 450NM. The dotted curves in Figure 5 represent the reliable window for the tolerance of 3.9NM but for which the  $v_{min}$  is increased with subsequent steps of 10kts. Thus it can be seen that the shape of the reliable  $ETA_{min} - ETA_{max}$  window, as well as its maximum and minimum values, depend heavily on the tolerance used and on the minimum speed the aircraft is able to fly.

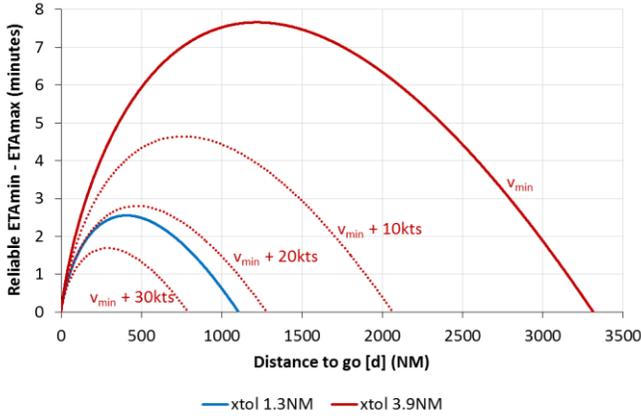


Figure 5. Reliable  $ETA_{min} - ETA_{max}$  window in function of distance to go, for various tolerances and minimum speeds.

Note that Figure 5 was derived assuming that the minimum speed of the aircraft remains constant. In reality the minimum speed will increase with aircraft weight. Thus as the distance to the constraint fix increases, the aircraft will be heavier and therefore the minimum speed will increase, which means that the shapes of the curves in Figure 5 are over-optimistic.

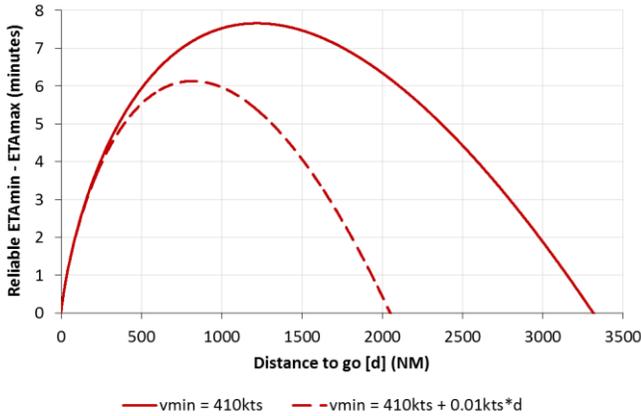


Figure 6. Reliable  $ETA_{min} - ETA_{max}$  window in function of distance to go, for constant minimum speed and minimum speed increasing with distance to go.

In Figure 6, the reliable  $ETA_{min} - ETA_{max}$  window for a tolerance of 3.9NM, a  $v_{max}$  of 470kts and a constant  $v_{min}$  of 410kts is compared with the function for the same conditions except that in the latter function  $v_{min}$  is variable. It was assumed that  $v_{min}$  would increase from 410kts with 0.01 knot per NM, which would mean 5kts of  $v_{min}$  increase over a distance of 500NM (roughly 1 hour of flight time). It can be seen that this

significantly affects the shape of the reliable  $ETA_{min} - ETA_{max}$  window.

## V. MINIMUM AND MAXIMUM OF THE RELIABLE $ETA_{min} - ETA_{max}$ WINDOW

Interesting to know is at which distance or time the reliable  $ETA_{min} - ETA_{max}$  window is at its minimum or maximum. Obviously the window is zero for a CTA equal to  $t_0$ . The window also becomes zero, or even negative if the uncertainty equals or exceeds the correction capability. If one assumes that the total required speed correction to achieve the reliable  $ETA_{max}$  and the total required speed correction to achieve the reliable  $ETA_{min}$  are of the same order of magnitude, in other words  $s(ETA_{max})$  approximates  $s(ETA_{min})$ , then the condition at which the reliable window becomes zero can be found as follows:

$$\begin{aligned} |ETA_{min} - ETA_{max}|_{rel} &= 0 \\ \Leftrightarrow \frac{d}{v_{min} + s} - \frac{d}{v_{max} - s} &= 0 \\ \Leftrightarrow s &= \frac{v_{max} - v_{min}}{2} \end{aligned} \quad (18)$$

The condition for which the reliable window reaches its maximum can be found by deriving equation (17) and equating it to zero. The derivative of  $s(ETA_{max})$  and  $s(ETA_{min})$  can be found by using equation (11) in which the CTA is replaced by respectively  $ETA_{max}$  and  $ETA_{min}$  and by using equation (12) to express  $ETA_{max}$  and  $ETA_{min}$  as a function of the distance to go  $d$  and the minimum and maximum speeds. In addition, it is again assumed that  $s(ETA_{max})$  is about equal to  $s(ETA_{min})$ , thus  $s(ETA_{max}) \approx s(ETA_{min}) = s$ . This yields the expression:

$$\begin{aligned} |ETA_{min} - ETA_{max}|_{rel} &= \text{maximum} \\ \Leftrightarrow \frac{d}{d(d)} (|ETA_{min} - ETA_{max}|_{rel}) &= 0 \\ \Leftrightarrow s &= \frac{v_{min} v_{max}^2 - v_{max} v_{min}^2}{v_{max}^2 + v_{min}^2} - w \\ \Leftrightarrow s &\cong \frac{v_{max} - v_{min}}{2} - w \end{aligned} \quad (19)$$

Equations (18) and (19) can be substituted back in equation (11) which allows finding the time to go to the CTA fix,  $CTA - t_0$ , for which the reliable  $ETA_{min} - ETA_{max}$  window gets zero, respectively maximal:

$$\begin{aligned} |ETA_{min} - ETA_{max}|_{rel} &= 0 \\ \Leftrightarrow CTA - t_0 &= \frac{x_{tol}}{w} \exp\left(\frac{v_{max} - v_{min}}{2w}\right) \end{aligned} \quad (20)$$

$$\begin{aligned} |ETA_{min} - ETA_{max}|_{rel} &= \text{maximum} \\ \Leftrightarrow CTA - t_0 &\cong \frac{x_{tol}}{w} \exp\left(\frac{v_{max} - v_{min}}{2w} - 1\right) \end{aligned} \quad (21)$$

This leads to an interesting observation: according to equations (20) and (21), the time to go to a CTA fix,  $CTA - t_0$ , for which the reliable  $ETA_{min} - ETA_{max}$  gets zero differs by a factor  $\exp(1) = e \approx 2.7$  from the time to go at which the

reliable  $ETA_{min} - ETA_{max}$  gets maximal (for a constant wind error and constant  $v_{min}$ ). In other words the time to go to the CTA fix for which the reliable  $ETA_{min} - ETA_{max}$  is at its maximum is about one third of the time to go to the CTA fix at which the reliable  $ETA_{min} - ETA_{max}$  is zero.

### VI. OPERATIONAL INTERPRETATION

The above theory has shown that taking into account a 10kts groundspeed uncertainty due to unknown meteorological conditions, a reliable  $ETA_{min} - ETA_{max}$  window of an aircraft increases with distance to go to a maximum and decreases to zero again thereafter. The distance at which the reliable  $ETA_{min} - ETA_{max}$  window becomes zero is the distance at which the speed buffers required to make speed corrections to compensate the wind uncertainty become as large as the operational speed window of the aircraft.

Figure 7 presents the graphical output of equation (21), indicating the time to go to the constraint fix at which the reliable  $ETA_{min} - ETA_{max}$  window gets maximal, as a function of the available true airspeed window  $v_{max} - v_{min}$  of the aircraft, for various control tolerances and an assumed wind error of 10kts. Note that the relation is expressed as a function of time to the CTA fix rather than distance and as such Figure 7 is valid for any wind condition. For example, if the speed limitations of an aircraft provide an available speed window of 80 KTAS and the control tolerance is 0.4NM (which would correspond to 10s at 140kts over the runway threshold), the maximum reliable  $ETA_{min} - ETA_{max}$  window will be at a remaining flight time of 0.8 hours or 48 minutes.

If the output from equation (21), graphically displayed in Figure 7, is multiplied by an average groundspeed, the distance at which the  $ETA_{min} - ETA_{max}$  window gets maximal can be estimated. This is displayed in Figure 8 for a wind uncertainty of 10kts, a control tolerance  $x_{tol}$  of 0.4NM (assuming a descent operation) and average groundspeeds of respectively 200, 300 and 400kts. For example, for an average groundspeed of 300kts with an available true airspeed window of 80kts, the distance to go at which  $ETA_{min} - ETA_{max}$  is maximal is 240NM. In case of a strong 100kts headwind or tailwind, this becomes respectively 160NM and 320NM.

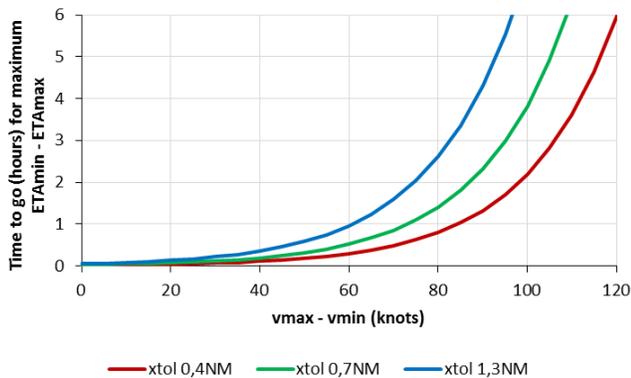


Figure 7. Time to go at which  $ETA_{min} - ETA_{max}$  window gets maximal as a function of speed window  $v_{max} - v_{min}$  and control tolerance  $x_{tol}$  with 10kts wind uncertainty.

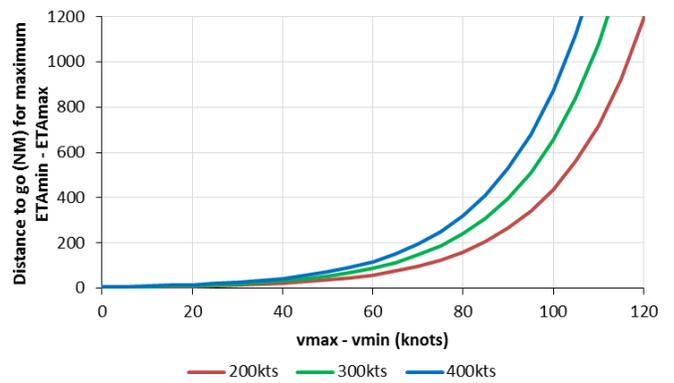


Figure 8. Distance to go at which  $ETA_{min} - ETA_{max}$  window gets maximal as a function of speed window  $v_{max} - v_{min}$  and average groundspeed with 10kts wind uncertainty and control tolerance 0.4NM.

Figures 5, 6, 7 and 8 indicate that the shape of the  $ETA_{min} - ETA_{max}$  window and its maximum are very sensitive to the available speed range  $v_{max} - v_{min}$  of the aircraft. The maximum operating speed of an aircraft is usually fixed and expressed as a maximum Mach/CAS schedule. The minimum speed however depends on altitude and aircraft weight. Figure 9 illustrates the difference between the maximum and minimum true airspeed as a function of flight level and assuming maximum landing weight, for 12 popular aircraft types. The data was collected directly from the aircraft's Flight Crew Operating Manuals.

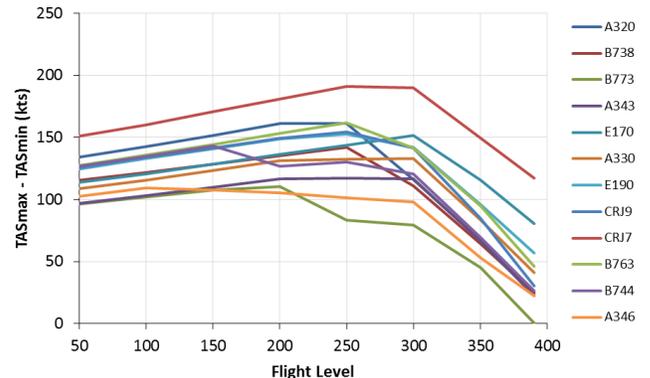


Figure 9. Difference between maximum and minimum True Airspeed versus Flight Level and aircraft type, for maximum landing weight.

As most jet aircraft cruise at relatively high flight levels (FL350-FL400), the true airspeed range can be considered to be below 100kts (typically between 50 and 80 knots), except for certain regional jet aircraft types like the CRJ7 and E170. The minimum speed of the latter aircraft is considerably lower which results in a higher speed range. An important factor that needs to be taken into account is the ATC acceptability of aircraft speed ranges. Air Traffic Controllers might not be aware of the low speed capability of certain aircraft types or might just not accept that some aircraft fly at speeds considerably lower than the nominal ones.

For these same aircraft, the dependence of the true airspeed range on aircraft weight is illustrated in Figure 10, applicable for a typical cruising altitude of 35,000ft.



equations (16) and (11) for an assumed wind uncertainty of 10kts and for two different control tolerances, being 0.7NM and 0.4NM. Figure 13 displays the range (difference between  $ETA_{max}$  and  $ETA_{min}$ ) of the  $ETA_{min} - ETA_{max}$  window as well as the range of the reliable  $ETA_{min} - ETA_{max}$  window for the two assumed control tolerances.

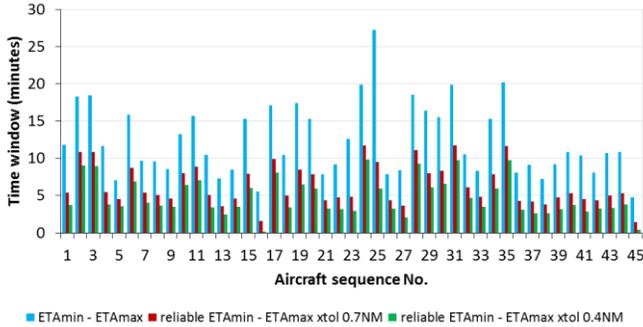


Figure 13. Range of  $ETA_{min} - ETA_{max}$  windows and reliable  $ETA_{min} - ETA_{max}$  windows assuming 0.7 and 0.4NM control tolerances, for 45 aircraft in Melbourne arrival sequence.

It can be observed from Figure 13 that the range of the reliable  $ETA_{min} - ETA_{max}$  window is mostly affected by the 10kts wind uncertainty. The fact that the aircraft needs to be able to make speed corrections to compensate this potential wind error, reduces the reliable  $ETA_{min} - ETA_{max}$  window to nearly half of its original size. The control tolerance has a secondary effect. More accurate arrival times, implying smaller control tolerances will lead to a smaller reliable  $ETA_{min} - ETA_{max}$  window. The control tolerance used for the remainder of this paragraph will be 0.7NM, which corresponds to 10s of time tolerance with a speed of 250 knots. This was chosen because existing Flight Management Systems providing RTA control do not yet control the speed in the final approach phase of flight. Therefore it is assumed in the further analysis that for CTAs at the threshold, the aircraft will control up to a point just prior to the start of the deceleration phase to final approach speed.

Starting the CTA operation at the extended AMAN horizon of 200NM, the study of [7] already highlighted the difficulty of getting all the proposed CTAs at the runway threshold in the achievable time window of the aircraft, ignoring meteorological forecast uncertainty. Reducing the achievable  $ETA_{min} - ETA_{max}$  windows to make them more reliable, as illustrated in Figure 13, would degrade the situation even further. Using equation (21), the orange dots in Figure 14 provide the time to go to the runway which yields a maximum reliable  $ETA_{min} - ETA_{max}$  window in function of the available true airspeed window of the aircraft, for the 45 aircraft in the recorded Melbourne arrival sequence. For some of the flights, the output of equation (21) gave a time to go that was larger than the total flight time. In this case the time to go was upper limited by the total flight time, which explains why not all of the orange dots in Figure 14 are on the solid orange curve representing equation (21). The green dots in Figure 14 represent the available speed window of the aircraft as a function of the cruise flight level, for all the 45 aircraft in the arrival sequence. Clearly there is a correlation between the flight level and the speed window. At the lower flight levels the

aircraft is able to fly a lower true airspeed and therefore the speed window is wider, which yields a greater time to go at which the reliable  $ETA_{min} - ETA_{max}$  window is maximal. Of course the latter applies when this time to go is not upper limited by the total flight time.

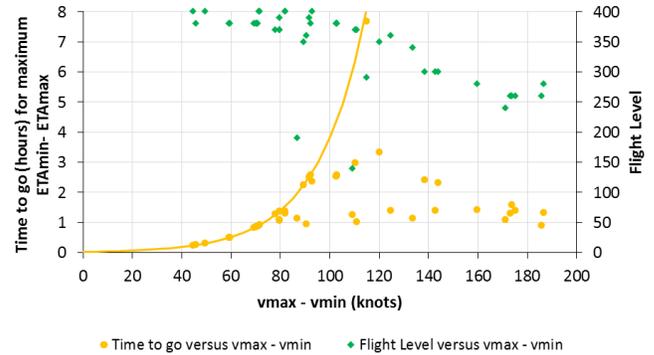


Figure 14. Time to go at which  $ETA_{min} - ETA_{max}$  is maximal versus speed window and speed window versus flight level for 45 aircraft in recorded Melbourne arrival sequence

Figure 14 indicates that for a lot of flights, the time to go for maximum reliable  $ETA_{min} - ETA_{max}$  has been limited by the total flight time. For those flights, the reliable  $ETA_{min} - ETA_{max}$  window is at its maximum at the take-off time. The majority of flights had a total flight time of less than 2 hours with lots of flights having a flight time of around 1 hour. Intuitively this suggests that even if a CTA is assigned further upstream of the 200NM extended AMAN horizon, the control range will still be limited due to the short overall duration of the flights.

In [7] two parameters were defined and used to indicate whether the assigned CTA at the threshold was in the aircraft's achievable time window and if not, how far the CTA was outside this window. Those parameters were:

$$Dev = \text{Max}(0, CTA - ETA_{max}) \quad (22)$$

$$X = \frac{CTA - ETA_{min}}{ETA_{max} - ETA_{min}} \quad (23)$$

$Dev$  represents the additional amount of time that needs to be lost after the application of a maximum speed reduction or in other words, the CTA minus the  $ETA_{max}$ .  $X$  represents the position of the CTA within the  $ETA_{min} - ETA_{max}$  window.  $X$  is defined as 0 if the CTA coincides with the  $ETA_{min}$ , 0.5 if the CTA is in the middle of the window and 1 or larger than 1 if the CTA is equal to or larger than the  $ETA_{max}$ .

Similar to the work presented in [7], Table I indicates for each aircraft in the arrival sequence whether the assigned CTA was in the achievable window (indicated in green) or outside this window (indicated in red), as well as the parameters  $Dev$  and  $X$  for three different cases: in the first case, the "real"  $ETA_{min} - ETA_{max}$  window, computed at the extended AMAN horizon (at 200NM), is used without considering meteorological uncertainty. The second case uses the reliable  $ETA_{min} - ETA_{max}$  window computed at extended AMAN horizon and the third cases uses the reliable  $ETA_{min} - ETA_{max}$  window computed at the time when this window is maximal. Wind uncertainty was considered to be 10kts. The control

TABLE I. YMML ARRIVALS 08 AUGUST 2012, 09-11 UTC, SIMULATION SCENARIOS APPLYING CTA AT 200NM WITH “REAL” VERSUS RELIABLE WINDOW AND APPLYING CTA AT TIME WHEN RELIABLE WINDOW IS MAXIMAL

CTA	Call sign	Type	Real ET <sub>Amin</sub> - ET <sub>Amax</sub> 200NM			Reliable ET <sub>Amin</sub> - ET <sub>Amax</sub> 200NM			Max. reliable ET <sub>Amin</sub> - ET <sub>Amax</sub>		
			CTA possible	Dev	X	CTA possible	Dev	X	CTA possible	Dev	X
08:59:42	VOZ866	B737			0,96		00:03:29	1,64		00:03:29	1,64
09:01:59	QFA451	B738			0,63			0,86			0,69
09:04:11	VOZ278	B738			0,85		00:02:40	1,25		00:01:11	1,09
09:06:14	VOZ868	B737		00:03:10	1,27		00:06:58	2,27		00:06:58	2,25
09:08:49	JST971	A320		00:00:31	1,07		00:01:45	1,39			0,95
09:12:13	JST451	A320			0,45			0,52			0,52
09:14:14	VOZ1377	B738		00:00:53	1,09		00:03:23	1,63		00:02:56	1,47
09:17:57	VOZ1331	B738		00:02:53	1,30		00:05:21	2,05		00:05:17	2,01
09:20:47	QFA476	B763		00:04:03	1,48		00:06:26	2,40		00:03:33	1,31
09:22:48	JST949	A320			0,82		00:00:43	1,09			0,71
09:26:28	QFA457	B738			0,57			0,75			0,65
09:28:14	VOZ870	B738		00:00:36	1,06		00:03:36	1,71		00:03:33	1,69
09:30:56	MAS129	A333		00:03:32	1,49		00:05:29	2,52		00:05:05	2,17
09:33:03	QFA692	B763			0,92		00:01:43	1,38		00:01:06	1,19
09:35:43	QFA631	B738			0,77		00:01:19	1,17			0,96
09:38:26	UAE407	B77W		00:04:33	1,82		00:06:12	4,95		00:06:12	4,95
09:41:04	VOZ874	B738			0,68			0,96			0,75
09:44:39	QFA459	B738		00:02:49	1,27		00:05:49	2,16		00:05:48	2,14
09:47:11	QFA833	B734			0,75		00:02:03	1,24		00:01:18	1,13
09:49:33	VOZ342	B738			0,63			0,90			0,80
09:51:20	QFA694	B738		00:03:04	1,39		00:04:57	2,14		00:04:40	1,96
09:55:29	TGW623	A320			0,64			0,70			0,70
09:57:46	RXA3683	SF34			0,70		00:01:24	1,29		00:01:21	1,24
10:00:42	JST479	A320			0,56			0,77			0,63
10:03:49	TFX152	SW4			0,58		00:01:46	1,18		00:01:41	1,17
10:05:07	VOZ236	B738		00:02:38	1,33		00:04:31	2,04		00:04:15	1,87
10:08:30	QFA455	A333		00:00:34	1,07		00:02:55	1,80		00:02:55	1,80
10:11:33	QFA463	B763			0,43			0,54			0,51
10:14:29	VOZ878	E190			0,53			0,79			0,73
10:16:08	QFA635	B738			0,54			0,70			0,59
10:19:14	JST525	A320			0,36			0,43			0,46
10:23:12	SQC7297	B744		00:00:44	1,07		00:03:12	1,53		00:00:41	1,06
10:26:08	JST712	A320			0,70			0,80			0,80
10:28:00	VOZ746	B738			0,67			0,97			0,85
10:30:16	QFA483	B734			0,32			0,38			0,40
10:33:09	CSN343	A332			0,67			0,83			0,75
10:36:16	VOZ882	B738			0,90		00:01:37	1,38		00:01:37	1,38
10:38:47	RBA53	B772		00:01:15	1,17		00:02:58	1,78		00:00:47	1,10
10:42:06	TGW631	A320		00:01:19	1,14		00:03:15	1,68		00:03:15	1,68
10:44:19	QFA465	B763			0,92		00:02:30	1,47		00:02:10	1,35
10:46:22	VOZ346	B738			0,57			0,71			0,71
10:48:30	QFA768	B763			0,54			0,65			0,64
10:53:00	QFA134	B738			0,81		00:01:09	1,23		00:01:07	1,22
10:56:03	QFA467	B763			0,68			0,97			0,93
11:00:04	QTR030	B77L			0,47			0,42			0,58
SUM				00:32:36			01:27:10			01:10:56	
AVG					0,86			1,33			1,20
STD					0,34			0,79			0,77

tolerance was set to 0.7NM. Note that although the same sequence was used as in [7], the individual values for the “real”  $ETA_{min} - ETA_{max}$  are not always exactly the same, as improvements were carried out in the simulation model and the aircraft performance (minimum and maximum speed) models. Table I indicates that it was much more difficult to get all the CTAs in the reliable achievable time window of the aircraft than in the case without considering meteorological uncertainty. More interesting is that if the CTAs were assigned when the  $ETA_{min} - ETA_{max}$  is maximal, this only resulted in a very modest improvement. Using the maximum reliable achievable time window, only three additional aircraft could reliably achieve their CTA by means of speed control. In this case, the total delay to be absorbed by other measures than speed control (expressed as the sum of the Dev parameters of all aircraft) went down from 01:27:10 to 01:10:56 (HH:MM:SS). Parameter X reduced from 1.33 to 1.20 which indicates that still for a high number of aircraft, it was not possible to get the CTA in the reliable achievable time window of the aircraft, even if the computation of this window was shifted upstream of the extended AMAN horizon where it achieved its maximum range.

### VIII. CONCLUSIONS AND FUTURE WORK

In this paper a model was presented that can be used to estimate the longitudinal uncertainty of an aircraft’s future trajectory while flying towards a Controlled Time of Arrival (CTA). The uncertainty is caused by the fact that the trajectory will be exposed to meteorological forecast errors and the guidance system will apply speed corrections to correct the predicted arrival time when a CTA needs to be met. Practically, this model can be used for the following purposes:

- Compute a longitudinal uncertainty window around an aircraft’s position while flying towards a CTA at any time between the start and end of the CTA operation. This can be used to assess the probability of spacing reductions between two in-trail aircraft performing a CTA operation, in the presence of meteorological uncertainty.
- Estimate the total magnitude of the speed corrections required to compensate the wind error to achieve a CTA within a set tolerance.

- Estimate by how much a predicted earliest-latest time window of an aircraft should be reduced so that any time within this reduced earliest-latest time window (also called the reliable  $ETA_{min} - ETA_{max}$  window) can be met with a high degree of certainty in the presence of an assumed maximum wind error.
- Estimate at which time during the flight this reliable  $ETA_{min} - ETA_{max}$  window becomes maximal, taking into account the assumed maximum wind error and a target control tolerance at the CTA fix.

The output of the analytical longitudinal uncertainty model was validated with the output from a real time CTA simulation using a B737 FMS workstation.

An analysis was performed for the time at which the reliable  $ETA_{min} - ETA_{max}$  window becomes maximal, for an assumed wind uncertainty and control tolerance. It was found that this time depends heavily on the available speed window of the aircraft, more in particular the minimum speed the aircraft is able to fly. This minimum speed also depends on the weight and altitude of the aircraft. A diagram was presented which determines the available speed range of 12 different aircraft as a function of altitude and weight, as well as the time to go at which the reliable  $ETA_{min} - ETA_{max}$  becomes maximal, as a function of the available speed range of the aircraft and for an assumed wind uncertainty and a set of control tolerances.

Finally as a practical test, the model was used to recompute the reliable  $ETA_{min} - ETA_{max}$  windows at the 200NM extended AMAN horizon for a recorded dataset consisting of 45 arriving aircraft to Melbourne airport. An earlier analysis performed in [7] concluded that it was not feasible to assign the recorded landing times as CTAs to the aircraft, as a large amount of the assigned CTAs were outside of the achievable time windows of the aircraft. The current study also computed the time to go to the CTA fix at which the reliable  $ETA_{min} - ETA_{max}$  window of each aircraft is maximal, as well as the magnitude of the reliable  $ETA_{min} - ETA_{max}$  window at this time. Assigning the CTA at the time when the reliable  $ETA_{min} - ETA_{max}$  is maximal did only yield a marginal improvement of the overall feasibility of the operation due to the fact that most aircraft in the arrival sequence have a relative short total flight duration and therefore the control range could not be extended drastically upstream of the extended 200NM AMAN horizon.

The results of this study suggest that if no other measures besides speed control (CTA) to achieve an arrival sequence are to be considered, earlier sequencing action must be taken with ultimately the departure time tactically adjusted. An area of further investigation could study the range of departure times as a function of total flight time, meteorological uncertainty and target control tolerance, required to achieve a CTA at the destination. Additionally it could be interesting to compute a reliable target time window for each aircraft at the extended AMAN horizon. If an aircraft arrives within this target time window it would be able to achieve its position in the landing sequence by solely speed control, even in the presence of an assumed meteorological uncertainty. Then the departure time window as well as the aircraft's cruising speed could be tuned, so that the aircraft would be able to arrive reliably within the target time window at the extended AMAN horizon, in the

presence of meteorological uncertainty but without the need to perform speed corrections during cruise. This would alleviate the burden of the arrival constraint further upstream of the flight, which would mean that the aircraft would be flying a constant, calculated speed in the en route ATC sectors, before arriving pre-conditioned within the target time window at the extended AMAN horizon. From there, the final CTA operation to the runway would start. In turn this supports CTA as one potential element in the toolset of traffic managers, however the operational applicability still needs to be further developed to efficiently process air traffic.

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## AUTHOR BIOGRAPHY

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