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A Simple Wake Vortex Encounter Severity Metric

Rolling Moment Coefficient due to Encounter of an Aircraft with a Wake Vortex

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***Abstract*—This paper presents a severity metric supporting the characterization of the effect of a wake vortex encounter on a follower aircraft. The proposed metric is the Rolling Moment Coefficient computed using various simplified assumptions that lead to a simple and usable metric. The metric can indeed be computed using available aircraft data. The paper also presents the assessment of the metric based on the results of a wake vortex encounter flight test campaign performed by Airbus. The use of the proposed metric yields good agreement with the experimental data. Because it is simple, yet realistic compared to experimental data, the metric can be used in a relative safety analysis permitting a reduction of the wake turbulence separations compared to today's ICAO standard, as was done in the framework of the RECAT-EU project.**

Keywords-Wake Turbulence; Wake Vortex Encounter; Rolling Moment; Severity Metric; Rolling Moment Coefficient; RECAT-EU

I. INTRODUCTION

As a consequence of its lift, an aircraft generates a complex turbulent wake, emanating from the wing and horizontal tail plane that rolls-up to form, in the far-field, a pair of counter-rotating vortices, as illustrated in Fig. 1, and lasting for several minutes after the aircraft has flown by. These vortices, whose initial circulation and lateral spacing depend on the aircraft characteristics, are transported and decay depending on the environmental conditions. The encounter of such a vortex pair by a follower aircraft can be hazardous due to the induced rolling moment and down-wash velocity. The wake encounter-related hazard is, together with the surveillance capabilities, at the origin of the separations to be applied between landing or departing aircraft. Nevertheless, the increase of air traffic in the recent years makes these separation standards a limiting factor for capacity at the busiest airports. A solution to airport congestion is thus to reduce, where possible, separation minima whilst at least maintaining the current safety level. For that purpose, a characterization of the behavior of wake vortices (WV) and of their interactions with a follower aircraft is needed.

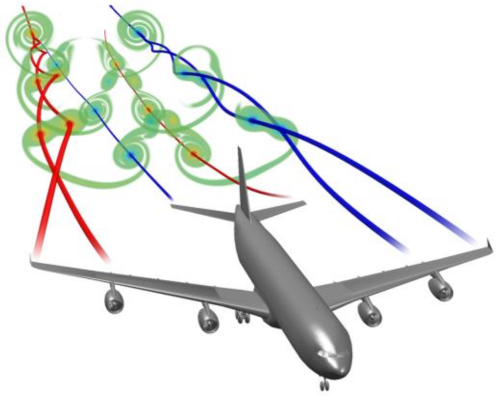

Figure 1. Illustration of the aircraft wake roll-up (Simulation by G. Daeninck, UCL)

A proposal, called "RECAT-EU" has been recently developed by the European Organization for the Safety of Air Navigation (EUROCONTROL). It consists of a revision of current ICAO provisions for wake turbulence separation minima on approach and departure leading to a proposed 6-category separation scheme, as opposed to the ICAO 3-category scheme[1]. The RECAT-EU proposal is supported by a safety case [1] that the European Aviation Safety Agency (EASA) confirms as providing the assurance that the RECAT-EU separation scheme can be used as basis to update current schemes [2].

One of the key elements in a proposal aiming at reducing separation is the severity metric used to quantify the wake vortex encounter (WVE) severity. The metric should take into account most of the WVE impact parameters such that it is realistic. However, it should only use quantities that are publically available or reasonably estimated such that it can be evaluated for all aircraft types. Finally the metric should allow direct comparison between various WVE cases (i.e., with various leader-follower aircraft pairs). The objective of the present paper is to detail one of the metrics that was used for the RECAT-EU proposal and its recent further improvements allowing an even better, yet still simple, characterization and quantification of the WVE severity.

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[1] Airbus A380-800 is not included in these 3 categories but covered by an ICAO State Letter recommending specific separations what de facto leads to a 4 category scheme

Work on WVE severity criteria has been on-going for several decades, see e.g. reviews in [3-8]. The metric that was used in RECAT-EU, and that is further improved in this paper, uses an existing and widely used severity metric that is the Rolling Moment Coefficient (RMC). This paper provides the simplified assumptions and the parameters to be able to calculate, a priori, the RMC related to a WVE for a given leader-follower aircraft pair.

The paper is organized as follows. Section II describes briefly the physics of wake turbulence. It also presents the effects on a follower aircraft of a WVE. Section III details the proposed severity metric. Finally Section IV shows the assessment of the proposed metric using the results of a WVE flight test campaign performed by Airbus.

II. WAKE TURBULENCE ENCOUNTER: DESCRIPTION AND IMPACT PARAMETERS

A. Wake vortex description

At a typical time of a WVE, the wake of an aircraft has rolled-up to form a two-vortex system (2VS) composed of a pair of counter-rotating vortices. The initial total circulation of the vortices, Γ_0, is related to the aircraft weight W_l, flight speed V_l, wing span b_l and wing and horizontal tail plane (HTP) loadings through:

$$\Gamma_0 = \frac{W_l}{\rho\, V_l\, s\, b_l} \ , \qquad (1)$$

with ρ the air density and s the spacing factor defined as the ratio between the initial lateral spacing between the vortices b_0, and the aircraft span, that depends on the combined effects of wing and HTP loadings.

Due to the mutually induced velocity, the vortex pair sinks and is also transported by the wind. The vortex total circulation, Γ_v, decreases in time, starting from Γ_0 and following a two-phase decay, with a decay rate that depends on the atmospheric turbulence and thermal stratification, see e.g. [9,10]. At lower altitude, because of the ground proximity, the descending vortices first move apart from each other, then interact with the ground and rebound, which also results in a decrease of their circulation (see description in e.g. [11]).

For new separation design, the reasonable worst case has been identified as the case of a follower aircraft flying at one wing span altitude when encountering the wake [12, 13]. At those altitudes, the WV are in ground effect (IGE). Therefore, and also in the rest of this paper, only the effect of one of the WV on the follower aircraft is considered; the other WV being considered as “far away”, due to the interaction with the ground. Note that, for WVE in altitude, the effect of a 2VS encounter and the associated metrics should be used (it can be obtained using an extension of the methodology proposed in this paper).

The circulation of the WV is distributed over a certain distance, following a function $\Gamma(r)$, with r the distance to the vortex center, and with $\Gamma(r)$ tending to the total circulation Γ_v for large r values. The vortex induced azimuthal velocity is then related to the circulation distribution through:

$$u_\theta = \frac{\Gamma(r)}{2\,\pi\, r} \ . \qquad (2)$$

B. Follower aircraft reaction

For the approach and landing phases, the WVE angles are typically low. The predominant impact of WV on the encountering aircraft is then a rolling motion [7, 8]. When encountering a single vortex aligned with its flight direction, the follower aircraft will roll, due to the vortex-induced rotational speed.

This roll effect is usually quantified by the rolling moment M_v. It can be measured during flight tests, see e.g. [14, 15]. It is then related to the roll acceleration, A_x, and to the rolling moment of inertia of the aircraft through:

$$M_v = A_x\, I_{xx}, \qquad (3)$$

with I_{xx} the rolling moment of inertia [kg m²] defined as

$$I_{xx} = M_f \left(\frac{b_f}{2}\, R_x\right)^2 \ , \qquad (4)$$

with, R_x the dimensionless radius of gyration [-], M_f the follower aircraft mass [kg] and b_f the follower wing span [m].

The rolling moment can also be computed using the WV characteristics. In what follows, without loss of generality, we consider the case of a counter-clockwise rotating vortex located at a position (y_v, z_v) w.r.t. the follower wing center, as illustrated in Fig. 2. The rolling moment is a consequence of a relative modification of the angle of attack along the wingspan. The local modification of the angle of attack is caused by the vortex induced vertical velocity $w_v(y)$ as seen by the aerodynamic profile at a position y, where y is the lateral position on the wing w.r.t. the wing center and related to the vortex circulation distribution. Because the angle of attack variation is small, it is well approximated through:

$$\begin{aligned} \Delta\alpha_v(y) &= \frac{w_v(y)}{V_f} \ , &(5)\\ &= \frac{1}{V_f}\,\frac{\Gamma(r)}{2\,\pi\, r}\,\frac{(y-y_v)}{r} \ , &(6)\end{aligned}$$

where V_f is the aircraft flight velocity, (y_v, z_v) the lateral and vertical position of the vortex w.r.t. the wing center, and r the distance to the vortex defined as:

$$r = \sqrt{(y-y_v)^2 + z_v^2}\,. \qquad (7)$$

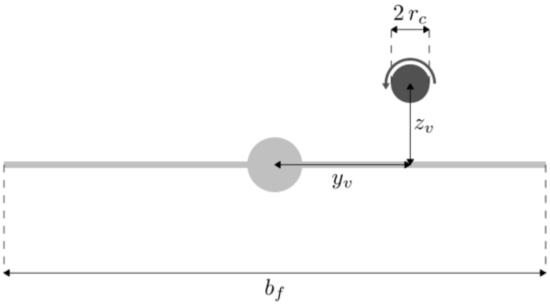

Figure 2. Scheme of the WVE geometry

The rolling moment induced by the vortex on the aircraft is then obtained as:

$$M_v = \frac{\rho V_f^2}{2} \int_{-\frac{b_f}{2}}^{\frac{b_f}{2}} C_{l_\alpha}(y)\, c(y)\, \Delta\alpha_v(y)\, y\, dy \,, \tag{8}$$

where *c(y)* is the wing chord distribution, and $C_{l\alpha}$ is the “effective lift slope coefficient” of the aerodynamic profile located at *y*, and here assumed as uniform along the span. Using (6), the rolling moment is then related to the wake position and circulation and to the follower aircraft characteristics through:

$$M_v = \frac{\rho V_f^2}{2}\, \bar{c}\, b_f\, \frac{\Gamma_v}{V_f}\, \frac{C_{l_\alpha}}{2\pi}\, \frac{1}{2} \int_{-1}^{1} \frac{c(\eta)}{\bar{c}}\, \frac{\Gamma(r)}{\Gamma_v}\, \frac{(\eta-\eta_v)}{((\eta-\eta_v)^2+\zeta_v^2)}\, \eta\, d\eta, \tag{9}$$

where $\bar{c}$ is the wing mean chord and where the following dimensionless quantities have been defined:

$$\begin{aligned} \eta &= \frac{y}{b_f/2}\,, \\ \eta_v &= \frac{y_v}{b_f/2}\,, \\ \zeta_v &= \frac{z_v}{b_f/2}\,. \end{aligned} \tag{10}$$

III. DEFINITION OF A SIMPLE METRIC FOR WAKE ENCOUNTER IN GROUND PROXIMITY

A. Rolling Moment Coefficient (RMC)

The proposed severity metric should allow direct comparison between various follower aircraft types. The rolling moment is, however, a dimensional quantity which makes it difficult to use for a global comparative analysis using various types of followers. The dimensionless Rolling Moment Coefficient (RMC) is then proposed as an appropriate severity metric. We note that the Roll Control Ratio (RCR) is another dimensionless rolling moment-based severity metric, using the roll control capability of an aircraft’s flight control system to normalize the rolling moment. However, since the roll control capability of an aircraft is an information not easily and not publicly available, the RCR does not constitute a suitable metric for the kind of studies we are interested in.

The RMC is a dimensionless rolling moment-based severity metric, accounting for the aircraft ability to recover. It is the rolling moment normalized using the aircraft flight speed, span and wing area S_f:

$$RMC = \frac{M_v}{\frac{1}{2}\,\rho\, V_f^2\, S_f\, b_f} \;. \tag{11}$$

Combining (9) and (11), the RMC is obtained through:

$$RMC = \frac{\Gamma_v}{V_f\, b_f}\, \frac{C_{l_\alpha}}{2\pi}\, \frac{1}{2} \int_{-1}^{1} \frac{c(\eta)}{\bar{c}}\, \frac{\Gamma(r)}{\Gamma_v}\, \frac{(\eta-\eta_v)}{((\eta-\eta_v)^2+\zeta_v^2)}\, \eta\, d\eta. \tag{12}$$

The circulation of a WV is distributed over a certain distance. According to [16], 50 to 60% of the circulation lies within a radius of 5% of the generator wing span; the rest of the circulation lies beyond that radius, and with a long tail in the distribution. In (12), the term $\Gamma(r)/\Gamma_v$ is hence solely a function of the generator wingspan b_l .The RMC is then obtained as a function of various dimensionless quantities:

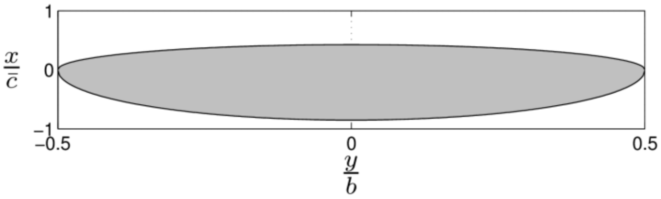

Figure 3. Top view of a wing with an elliptical chord distribution.

$$RMC = \frac{\Gamma_v}{V_f\, b_f}\, \frac{C_{l_\alpha}}{2\pi}\, F_{\text{corr}}\left(\frac{b_l}{b_f}, \frac{y_v}{b_f}, \frac{z_v}{b_f}, \frac{c(y)}{\bar{c}}\right) \;. \tag{13}$$

Note that the latter equation could have been obtained simply by dimensional analysis without any assumption on the WVE.

B. Follower aircraft characteristics

For the sake of simplicity, we here consider that the follower wing chord follows an elliptical distribution described as:

$$\frac{c(y)}{\bar{c}} = \frac{4}{\pi}\sqrt{1-\left(\frac{y}{b/2}\right)^2} \;. \tag{14}$$

The chord distribution is illustrated in Fig. 3. Note that in RECAT-EU [1], various wing chord distributions were considered. The sensitivity of the results to the chord distribution was however shown to be low.

C. Circulation distribution model

As explained here above, the circulation of a WV is distributed over a certain distance, itself scaling with the leader wingspan. Various circulation distribution models are found in literature. All models describe the circulation evolution as a function of the distance *r* from the vortex center and assume an axisymmetric vortex. We make a distinction between two classes of circulation distribution models. The first class concerns circulation distribution models with a “core parameter” that have a velocity equal to zero at *r=0*, then increasing to reach a maximum at the “effective core radius” and then decreasing. Examples are the Burnham-Hallock (B-H) model [17] (also called the “Low Order Algebraic” model, or the “Rosenhead” model), the Lamb-Oseen model or the High-Order Algebraic model. The second class concerns the models without core that have a monotonic decreasing velocity profile, and thus an infinite velocity at *r=0*; yet they have a finite energy for the vortex pair (i.e. energy of the 2VS), as it should. Because of its simplicity and because it is often used in the WV literature, we use the B-H model. It is defined as:

$$\frac{\Gamma(r)}{\Gamma_v} = \frac{r^2}{(r^2+r_c^2)} \;, \tag{15}$$

with r_c the core parameter (and which is here also the effective core size, i.e., the radius of maximum induced velocity), and taken as a percentage of the generator aircraft span. We stress that metrics using circulation profiles without core can also be developed and provide essentially identical RMC results, so long as we use models so that the energy of the obtained 2VS is the same, see, e.g., [18].

D. Effective lift slope coefficient

The “effective lift slope” to use in the rolling moment evaluation must be diminished compared to the true “profile lift slope”, to account for the effect of the additional vortex wake generated by the encounter itself. Because of the influence of the wake, shed by the follower aircraft, on the effective angle of attack, the “effective lift slope” of each wing section, $C_{l,\alpha}(y)$, is indeed not uniform. It is also not equal to the lift slope of the aerodynamic profile (itself typically assumed equal to 2π). It is recognized that this effect is quite complex, even for a wing in level flight alone; and that is even more complex for the case of a wing with a WVE. For an elliptical wing in level flight the effective lift slope is found to be uniform along the span and to correspond to the aerodynamic profile lift slope multiplied by $AR_f / (AR_f+2)$, with AR_f the wing aspect ratio. This is the well-known “Prandtl correction”. This correction is no longer valid for the case of a wake encounter. The proposition retained here is to take this effect into account in a global way: by correcting, a posteriori, the metric obtained assuming a profile lift slope equal to 2π, and using a correction factor that also only depends on the wing aspect ratio:

$$C_{l\alpha} = 2\,\pi\,\frac{AR_f}{AR_f + C}\ , \qquad (16)$$

with C a parameter still to be determined. For the case of a centered encounter (i.e., with the follower aircraft wing center exactly in the vortex center), because half of the wing experiences an up-wash velocity whereas the second half experiences a down-wash velocity, Rossow [8] stated that the left and right wing-halves should be considered separately. The effective $C_{l\alpha}$ value should therefore be computed using the aspect ratio based on the half wing. Then, using the Prandtl correction and the half wing aspect ratio is equivalent to the use of a value $C=4.0$ in (16).

The C value can also be obtained by solving the Prandtl integral equation for the case of a wake encounter. The “perturbation span loading” corresponding to a WVE is obtained by solving numerically the Prandtl integral equation for the case of a steady encounter with a B-H vortex. The “perturbation effective lift slope" corresponding to the encounter effects is then also obtained, along the span. It is obtained not uniform across the span. The “a posteriori and global" correction function is then calibrated on the results. For an elliptical wing chord distribution, a value of $C=4.0$ is found for aspect ratios values ranging from 7 to 15 with a vortex core parameter equal to 5% of the follower span, see [19]. This value of $C=4.0$ was confirmed to be robust when changing the WVE parameters. $C=4.0$ is indeed obtained for aspect ratios values ranging from 5 to 20 and for vortex core parameters ranging from 2% to 8% of the follower span. Furthermore, the value $C=4.0$ is also obtained for the case of non-centered WVE.

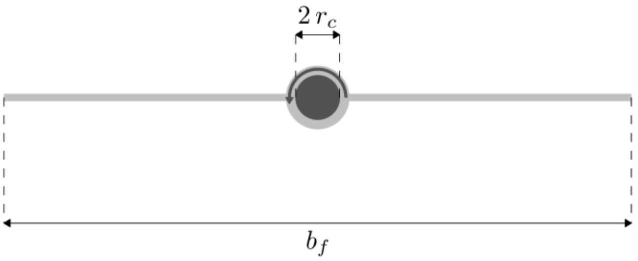

Figure 4. Considered WVE topology

E. Encounter topology

The maximum induced rolling moment is obtained when the vortex center is aligned with the follower wing center. It is therefore the situation considered for the metric and is illustrated in Fig. 4.

Considering the case of the encounter by a follower, having a wing with elliptic chord distribution, with a vortex, that has a B-H circulation distribution, and that is centered on the follower wing, the RMC expressed in (13) is then only a function of the span ratio b_l/b_f.

It is obtained as:

$$RMC = \frac{\Gamma_v}{V_f\,b_f}\frac{AR_f}{(AR_f + C)}\,G\left(\epsilon_v\right), \qquad (17)$$

where $\varepsilon_v=(2\ r_c)/b_f$ is a function of the span ratio b_l/b_f (see below) and G is the correction function, illustrated in Fig. 5, that reads:

$$G(\epsilon_v) = 1 - 2\,\epsilon_v\left(\sqrt{1+\epsilon_v^2} - \epsilon_v\right). \qquad (18)$$

In obtaining (18), the aircraft fuselage is simply considered as an extent of the wing.

F. Vortex core parameter

In the RECAT-EU metric, the core parameter (= “effective core size”) r_c in the B-H vortex model is taken as 4% of the leader span b_l. This choice was based on literature, where the typical values used range between 3% and 5% of the generating aircraft.

The value of the core parameter to be used in the B-H model can also be obtained from an energy-based analysis using the

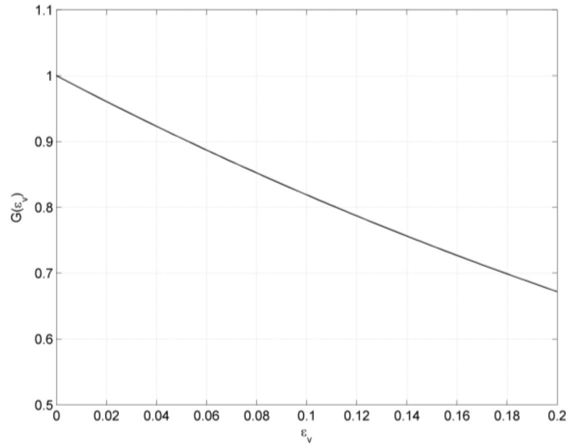

Figure 5. Correction function accounting for the size of the vortex compared to the follower

lift distribution of the leader aircraft. The argument is based on the fact that the kinetic energy of the near wake cross-flow, E_0, cannot be lower than that of the rolled up 2VS, E_v. The cross-flow kinetic energy, when multiplied by the air density, is also equal to the induced drag, itself related to the lift and hence to the circulation distribution. The cross-flow kinetic energy of the near-wake is then obtained as:

$$E_0 = \frac{s^2}{e}\frac{2}{\pi}\Gamma_0^2 \ , \tag{19}$$

where s is the spacing factor and e the Oswald efficiency. Using the classical Prandtl lifting line theory, s and e can be obtained from the circulation distribution $\Gamma(y)$. The energy of a 2VS made of B-H vortices can also be computed using [20]:

$$E_v \ = \ \frac{\Gamma_v^2}{2\,\pi}\left(\log\left(\frac{b_v}{r_c}\right) - \frac{1}{2}\right), \tag{20}$$

where Γ_v and b_v are the total circulation and the vortex spacing of the 2VS. Considering the case of a young rolled-up 2VS: as it hasn't decayed yet, one has that $\Gamma_v = \Gamma_0$ and $E_v = E_0$. By momentum conservation, one also has that $\Gamma_v\, b_v = \Gamma_0\, b_0$. The effective core size is then finally obtained as:

$$\frac{r_c}{b} = s\,\exp\left(-\left[4\frac{s^2}{e} + \frac{1}{2}\right]\right). \tag{21}$$

Various span-loading models are considered here. First, we consider the hyper-elliptic lift distribution family, expressed by:

$$\Gamma(y) = \Gamma_0\,\left(1 - \left(\frac{|y|}{b/2}\right)^p\right)^{1/p}. \tag{22}$$

Using $p=2$, (22) leads to the classical elliptic lift distribution. The cases with $p=2.5$ and $p=3.0$ are also investigated as representative of "more rectangular" wings. The three investigated hyper-elliptic circulation distribution are provided in Fig. 6.

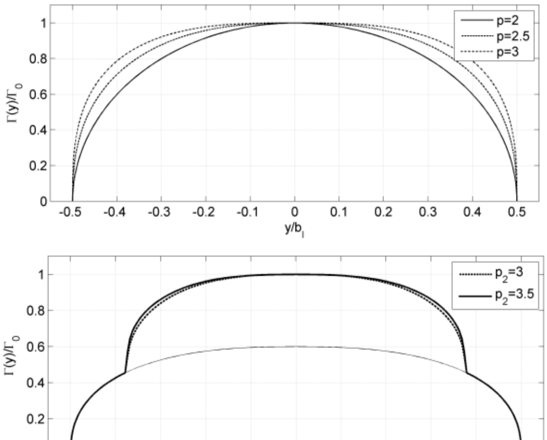

Figure 6. Considered circulation distributions. Top: Hyper-elliptic distributions; Bottom: Double hyper-elliptic distributions

TABLE I. SPACING FACTOR, OSWALD EFFICIENCY AND EQUIVALENT VORTEX B-H CORE PARAMETER FOR VARIOUS CIRCULATION DISTRIBUTIONS

Circulation distribution	**s**	**e**	**r_c/b_l**
Elliptic (p=2)	π/4	1.00	4.04%
Hyper-elliptic (p=2.5)	0.85	0.98	2.76%
Hyper-elliptic (p=3)	0.88	0.93	1.89%
Double hyper-elliptic (p_1=2.5, p_2=3)	0.77	0.94	3.66%
Double hyper-elliptic (p_1=2.5, p_2=3.5)	0.78	0.93	3.42%

For the elliptical lift distribution, $s=\pi/4$ and $e=1.0$, applying (21) leads to an equivalent B-H vortex core parameter equal to 4% of the generator span. For the cases with $p=2.5$ and $p=3.0$, one obtains $r_c/b=2.8\%$ and $r_c/b=1.9\%$.

In approach configuration, the wing loading is more realistically represented using a double hyper-elliptic lift distribution. In addition to the lift distribution of the "clean" wing, this model accounts for the lift distribution of the deflected flaps, extending over a fraction α of the wing span, and responsible for the production of $(1-\beta)$ of the total circulation. The circulation span-wise distribution model then reads:

$$\frac{\Gamma(y)}{\Gamma_0} = \begin{cases} \beta\,\left(1-\left(\frac{|y|}{\frac{b}{2}}\right)^{p_1}\right)^{1/p_1} & \text{if } |y| < \alpha\,\frac{b}{2} \\ \beta\,\left(1-\left(\frac{|y|}{\frac{b}{2}}\right)^{p_1}\right)^{1/p_1} + (1-\beta)\,\left(1-\left(\frac{|y|}{\alpha\,\frac{b}{2}}\right)^{p_2}\right)^{1/p_2} & \text{else.} \end{cases} \tag{23}$$

As typical values, we here use $\beta=0.6$ and $\alpha=0.75$, $p_1=2.5$ and $p_2=3$ or 3.5 (as the flap is more rectangular than the wing). The two double hyper-elliptic circulation distributions are shown in Fig. 6. A value close to $r_c/b=3.5\%$ is found for both cases. Table I provides the spacing factor, the Oswald efficiency and the equivalent B-H vortex core parameter for the investigated circulation distributions.

The energy of a near wake can also be obtained using wind tunnel measurements as those collected by Airbus during the European FP5 AWIATOR research project, see [21]. In this project, the near-wake velocity field of an A340 in landing configuration was indeed measured. From those measurements, the vorticity and stream function can be computed and integrated to obtain the cross-flow energy. Equating the obtained energy with (20) leads to a core parameter of 4 to 4.5% of the leader span. However, it should be noted that the measurement resolution (170 points per wingspan) was not sufficient to capture strong velocity gradients. The obtained vorticity field is hence artificially diffused when computed, as compared to reality. With a better resolution, a higher energy value is expected and consequently a slightly smaller effective core size value for the B-H model vortices. Therefore, a value of r_c/b_l of 3.5% is considered from now on for the proposed metric.

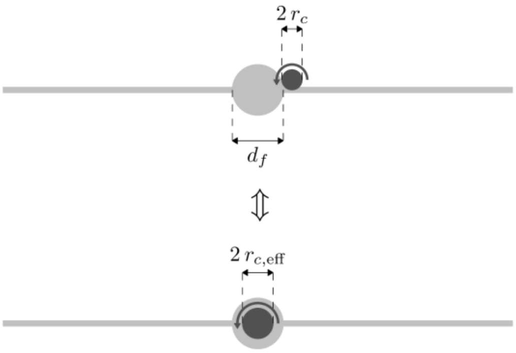

Figure 7. Illustration of a non-centered WVE and of the concept of effective core size.

Finally, it is also to be noted that, the effect of the encounter with a WV of core size r_c, that is not centered on the follower wing, is equivalent to the effect of the encounter with a vortex, centered on the follower wing, and with an "effective core size of encounter", $r_{c,eff}$, that is larger than r_c. This is illustrated in Fig. 7.

For instance, if one considers the RMC induced on a follower, by a vortex "touching" the follower wing and fuselage (e.g., a vortex positioned at a distance r_c from the fuselage and the wing), see illustration in Fig. 7, the metric to use is then that using the formula for a centered encountered but with a dimensionless "effective core size of encounter". This effective core size of encounter is obtained from the dimensionless vortex core size, and the ratio between fuselage diameter and wing span $\varepsilon_f=d_f / b_f$ so that the induced RMC are equivalent, see illustration in Fig. 7. Considering a typical fuselage diameter of 10% of the follower span, the effective core size of encounter is satisfactorily approximated by:

$$\epsilon_{v,\mathrm{eff}} = 0.0098 + 1.64\,\epsilon_v\,. \qquad (24)$$

The evolution of the effective core size of encounter is provided in Fig. 8.

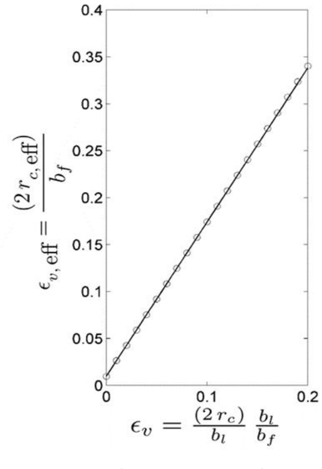

Figure 8. Effective core size of encounter as a function of the vortex core parameter for ε_f=10% (symbols) and linear approximation of the evolution (solid)

G. *Wake vortex circulation*

A major input required to calculate the RMC is the WV circulation at time of encounter. This quantity can be obtained from various sources. First, it can be obtained (at least approximately) by processing LiDAR velocity measurements.

In RECAT-EU, the circulation, in reasonable worst case (defined as the top 2% of the longest lasting wakes and that constitutes the design case, benchmarked against the measurements campaigns used for the A380 and B747-8 safety cases [12, 13]), was obtained using the initial 2VS characteristics (i.e., b_0 and Γ_0) and a generic dimensionless circulation decay curve expressing Γ_v/Γ_0 as a function of t/t_0, where $t_0=b_0/V_0$ is the characteristic time of the 2VS, with $V_0=\Gamma_0/(2\pi\, b_0)$ the 2VS sink rate. This generic decay curve was built using the heavy-generated WV LiDAR measurements, collected in London for two years (EGLL-1 campaign). The dimensionless decay curves for each aircraft type were indeed found to collapse on a single curve, defined as the generic dimensionless decay curve.

Finally, note also that, in the absence of WV measurements, the WV circulation evolution can also be obtained using WV prediction tools, such as the Deterministic wake Vortex Model (DVM) [11]. The DVM, which has been developed at UCL, uses several simplified physics-based models so as to forecast the transport and circulation decay of WV, depending on the aircraft that generates them and on the environmental conditions (meteorological conditions and ground proximity). The WAKE4D platform [22], which uses the DVM as a sub-tool, was used to support the acceptance, by DGAC, of the WIDAO concept at Paris-Charles de Gaulle airport [23].

IV. ASSESSMENT OF THE METRIC

A. *Reference Data*

Airbus flight test data, performed in 2011, are used for the assessment of the proposed metric for the induced rolling moment. The Airbus data include wake encounters in landing configuration involving two heavy generator aircraft (i.e., A380 and A346) and two different follower aircraft (A343 and A320), see [14, 15]. The database contains, amongst other quantities, the follower flight speed, the air density, the measured roll acceleration and the corresponding computed rolling moment, the estimated minimum distance to the WV core obtained using Wake-ID [24], and the estimated WV circulation. A manual quality screening of the data was also conducted by an independent expert panel, in the framework of the A380 ICAO pilot working group led by EASA. Furthermore, the database also contains the pilot encounter descriptions recorded during the flight test. The data let us compute the measured RMC and compute the RMC metric using the assumption proposed in this paper.

A subset of the wake encounter data was selected for further analysis on the basis of three criteria:

- Data with quality rating judged sufficient by the independent expert panel,

- Data for “almost centered” encounters (i.e., with a distance to the vortex core below 0.25 of the follower span) ,
- Data for which the measured circulation is reasonable compared to the aircraft type (i.e., value below the estimated initial circulation computed using (1)).

B. Metric assessment

In order to assess the quality of the proposed severity metric, three “variations” of the RMC are considered here:

- in metric 0, the RMC is simply defined as the ratio between the vortex circulation Γ_v and the product of the follower flight speed and span $(V_f \times b_f)$;
- the RECAT-EU metric considering the RMC of a centered WVE with a B-H vortex, with a core parameter of 4% of the leader span, by a follower with an elliptical chord distribution and with an effective lift slope coefficient of $2\pi AR_f/(AR_f+2)$;
- the RMC metric proposed here, that considers a centered WVE with a B-H vortex, with a core parameter of 3.5% of the leader span, by a follower with an elliptical chord distribution and with an effective lift slope coefficient of $2\pi AR_f/(AR_f+4)$.

The three considered metric are therefore expressed as:

$$RMC_0 = \frac{\Gamma_v}{V_f\, b_f} \quad , \tag{25}$$

$$RMC_{\text{R-EU}} = \frac{\Gamma_v}{V_f\, b_f}\,\frac{AR_f}{AR_f+2}\,G\left(\epsilon_v = \frac{0.04\, b_l}{b_f/2}\right), \tag{26}$$

$$RMC_{\text{Proposed}} = \frac{\Gamma_v}{V_f\, b_f}\,\frac{AR_f}{AR_f+4}\,G\left(\epsilon_v = \frac{0.035\, b_l}{b_f/2}\right). \tag{27}$$

The results of metric 0 are provided so as to compare the RMC valuation obtained using the corrected metrics to that obtained when solely using the leading order term of (13).

Fig. 9 to 11 show the correlation between the encountered WV circulation and the rolling moment (as experienced and measured, and as obtained from re-dimensionlization of the metric result). One observes that, for a given follower, the rolling moment is strongly related to the WV circulation. However, without correction function, this circulation dependence is artificially increased, as observed in Fig. 9.

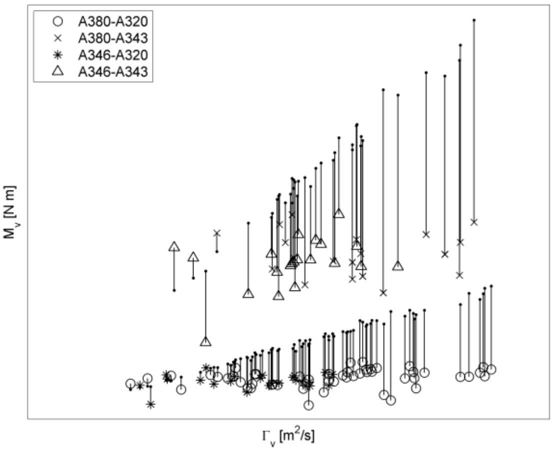

Figure 9. Rolling moment versus measured vortex circulation: measured rolling moment (symbols) and that obtained using metric 0 (dots).

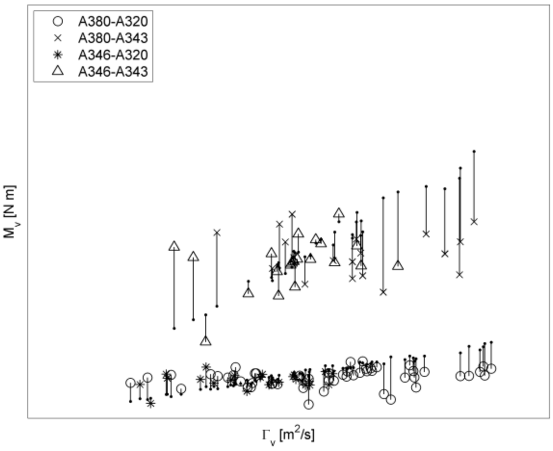

Figure 10. Rolling moment versus measured vortex circulation: measured rolling moment (symbols) and that obtained using RECAT-EU metric (dots).

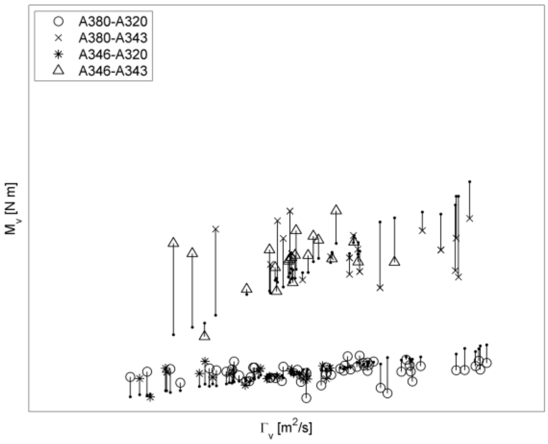

Figure 11. Rolling moment versus measured vortex circulation: measured rolling moment (symbols) and that obtained using the proposed metric (dots).

Figs. 12 to 14 compare the various correlations obtained between the measured rolling moments and those obtained from re-dimensionalization of the RMC metrics. Compared to the use of the metric 0, one observes a much better correlation when using the RECAT-EU metric, and an even further improved correlation when using the metric proposed here.

Figs. 15 and 16 provide the correlation between the measured RMC and those from the RECAT-EU metric and from the proposed metric. The correlation between measured and obtained RMC is seen to be good for both metrics, yet the proposed metric yields an even better correlation.

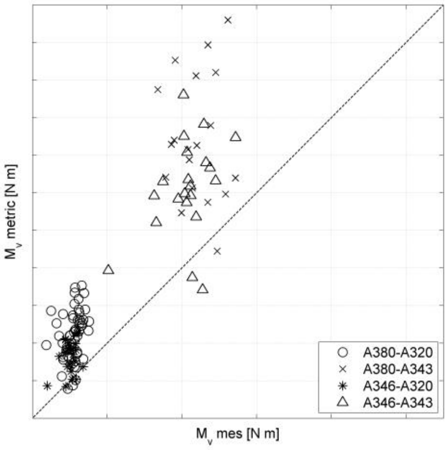

Figure 12. Rolling moment obtained using the metric 0 versus measured rolling moment.

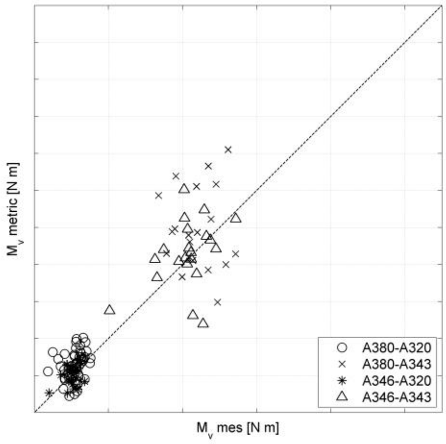

Figure 13. Rolling moment obtained using the RECAT-EU metric versus measured rolling moment.

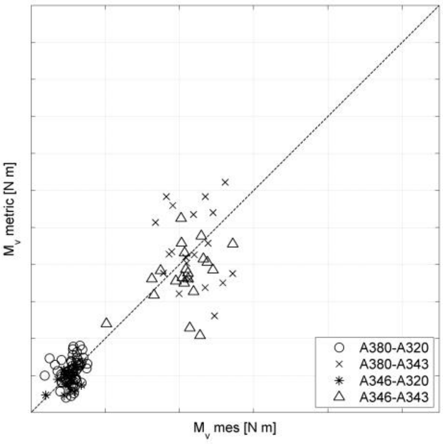

Figure 14. Rolling moment obtained using the proposed metric versus measured rolling moment.

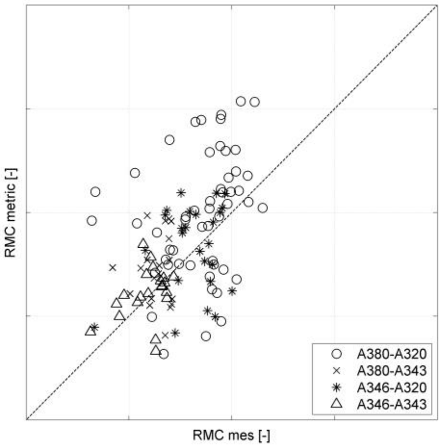

Figure 15. RMC obtained using the RECAT-EU metric versus measured RMC.

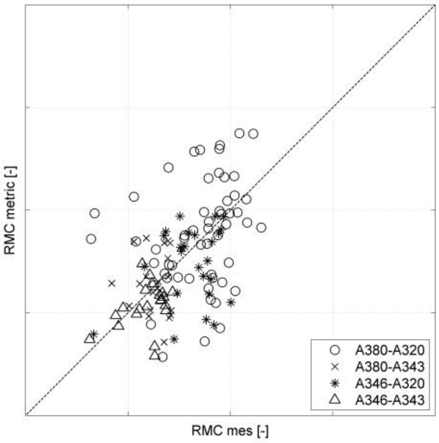

Figure 16. RMC obtained using the proposed metric versus measured RMC.

TABLE II. OBTAINED STATISTICAL DIFFERENCES BETWEEN THE METRICS AND THE EXPERIMENTAL DATA

Metric	**RMC mean deviation**	**RMC rms deviation**	**Best fitted linear coefficient**	**R^2 of the linear fit**
Metric 0 : $\Gamma/(V b_f)$	0.0635	0.0777	1.71	0.77
RECAT-EU Metric	0.0087	0.0262	1.12	0.82
Present work	-0.0009	0.0226	0.98	0.82

Finally, Table II provides the statistics of the correlation between the measurements and the severity metrics. On average, the proposed RMC severity metric has a very small bias (close to zero) compared to the measurements, whereas the RECAT-EU metric slightly over-estimates the RMC (small bias) and the metric 0 significantly over-estimates it (large bias). The RMC rms deviation is lowest when using the proposed metric, and the best fitted linear coefficient between the measured and the calculated RMC is close to 1 with a coefficient of determination $R^2=0.82$. This correlation is improved compared to the use of the RECAT-EU metric.

V. CONCLUSION

A simple metric supporting the characterization of the wake vortex encounter (WVE) severity has been proposed. This metric was developed using various simplified assumptions that were justified and that lead to a simple and usable metric. The proposed metric is the Rolling Moment Coefficient (RMC) for the case of an encounter, by a follower aircraft, having an elliptical wing chord distribution, with a wake vortex, that is centered on the follower wing and that has a Burnham-Hallock circulation distribution with an effective core parameter equal to 3.5% of the generator wingspan. The effective lift slope coefficient of the follower wing is also modified to account for the effect of the WVE: a simple and global correction is used, and that solely depends on the wing aspect ratio. The proposed metric was assessed based on the results of a WVE flight test campaign performed by Airbus. The use of the proposed metric yields good agreement with the experimental data. The correlation with the experimental data is even further improved compared to what is observed when using the RECAT-EU severity metric.

Because it is sufficiently simple to be calculated for any aircraft pair, yet sufficiently accurate, the proposed WVE severity metric constitutes a good candidate to be used as a tool for WVE risk analysis related to a modification of the wake turbulence separation rules.

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