National Airspace Capacity Estimate

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Abstract

Since the air traffic process has persistence, any air traffic model should address persistence and the associated dependency among events. For example, the airborne count is persistent and an hourly distribution of the airborne count can be incorporated into a non-linear model to estimate the carrying capacity of the airspace.

Introduction

When attempting to model something as dynamic and complex as the national airspace system (NAS), it is important to consider whether the models and assumptions used truly reflect the underlying realities of the system. Countless models have been developed to answer questions about the current and future performance of the national airspace system. For example, model the throughput of a sector or design an airport. In recent years, decision-makers have tended to ask "bigger" questions. Such as, what is the proper level of investment in air traffic management technology, or how close are we to reaching capacity? Advances in computing power and software engineering have made it comfortable for researchers to attack such questions by stringing together existing models that were intended to accomplish smaller tasks. However, this may not be the most appropriate approach. Before models are cut and pasted together, the long-forgotten assumptions that drive these models need to be examined and compared with both intuitive and empirical knowledge of the air traffic system. Once this examination is done, it is possible to consider mathematical approaches proven in other disciplines to make better models and more effective predictions.

The following discussion demonstrates this approach by attempting to estimate the capacity of the NAS. This estimate is derived through re-examination of the underlying dynamics of the NAS and the application of a simple equation whose complex behavior in many ways reflects the complex dynamics of the NAS.

Air Traffic Management Process and Persistence

The air traffic management process is complex in both the conventional sense and the mathematical sense. The air traffic management system must ensure the safe and orderly flow of air traffic, and at the same time, allow commercial operators the flexibility they need to effectively manage their economic assets. Thousands of people engage in this process every day. They look at the system from their own perspectives often using their own data. They make assumptions about what the other participants in the system will do, and they develop strategies that they hope will influence the overall performance of the system and serve their best interests. All of these participants are forced to balance competing interests. For example: a restriction imposed by one air traffic facility almost always places a burden on another, overall efforts to limit demands to safe levels come at the cost of efficiency, and schedule adjustments that make money for one airline often place additional costs on several others.

It should be no surprise that the data generated by such a system shows evidence of mathematical complexity. The net effect of all of these independent participants shows up as persistence in the flow if aircraft. For example, the delays associated with a particular flight are persistent [Cocanower, 1997].

Another aspect of persistence in the context of real world processes is that the sequential order for dependent events is significant. And, time provides the basis for ordering the dependent events and preserving the dependency. In fact, time is a continuously increasing variable[1] for these events [page 293, Prigogine 1984]. Hence, any comprehensive model of a process with persistence needs to acknowledge that associated events can be dependent and that those events can be ordered with time as a continuously increasing variable.

[1] One of the limitations of classical mechanics and of quantum mechanics is that they allow symmetric time reversals [Prigogine 1993]. Prigogine has proposed extensions to classical mechanics and to quantum mechanics to overcome this limitation.

Airspace Model

Based on the preceding discussion, a proper mathematical model of the NAS should have certain characteristics. It should reflect that: 1) the operation of the NAS is the net result of countless participants taking action based on limited information in a competitive environment; 2) there is persistence in that what happens one day, one month, or one hour has a bearing on what follows.

Existing models of the NAS do not fit this description well. They normally apply simple linear equations to allocate aircraft to queues of a certain capacity. They do little to reflect the actions and adaptations of air traffic managers and airline operators that are central to the overall performance of the system. Likewise, current models largely ignore the aspect of learning or persistence. The world is “reborn” at the beginning of each simulation. Aircraft rediscover each constraint as if it were new even though it may have bumped into the same constraint a hundred times before.

A mathematical model that does seem to apply to this problem is the logistics equation (See Appendix). This equation was developed to estimate the change in population in a colony of living creatures. It addresses some of the real-world logistics issues associated with the maintenance of a population. This equation may seem like an odd choice for an airspace model but consider the similarities. Populations compete for finite resources such as food and territory in much in the same way as airlines compete for capacity. Populations are subject to disease and predators, and in time develop strategies for dealing with these circumstances much in the same way that the participants of the NAS learn to deal with weather and other disruptions.

The logistics model also deals with the issue of persistence. The logistics equation is linked to the birth, life, and death cycle of a population of creatures. Other systems can have a similar birth-life-death cycle such as the departure, flight, and arrival phases of the air traffic control process. The analogy continues with the number of aircraft aloft being equivalent to the population of the colony of living creatures [page 1, Kingsford 1988]. In other words, the change in the number of aircraft aloft or instantaneous airborne count (IAC) is a candidate for a model based on the logistics equation. In the logistics model, events in one generation impact the next generation, just as events for one bank of flights has an impact on the next bank.

The Appendix describes some additional conditions for selecting the logistics equation as a model. First, the IAC values are discrete values with some uncertainties based on several variables including airline schedules, air traffic procedures, and weather. Second, the IAC values tend not to overlap since the sampling interval is comparable to the duration of a flight. Third, each airborne aircraft has an existence of its own and is not the product of two parent aircraft.

While the conditions for tailoring the logistics equation to the airspace process are reasonable, the logistics equation should be considered as an initial step in the development of a non-linear model for an airspace with persistent characteristics.

ETMS Data

The input for an airspace model based on the logistics equation is the hourly distribution of IAC values derived from Enhanced Traffic Management System (ETMS) data (See Figure 1).

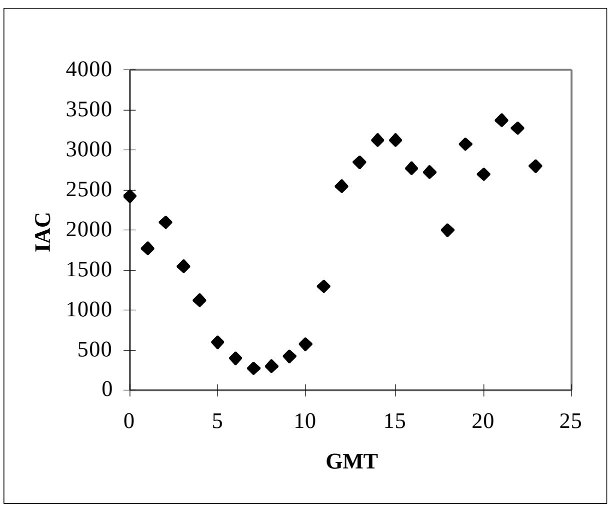

Figure 1: US IAC Distribution, 13 March 1997

The first step in the process of applying the logistics equation to the air traffic process is to prepare a difference diagram for the IAC distribution. That is, each plotted point in the difference diagram has the coordinates, IAC(i) and IAC(i+1), where "i" is an index in the time series of IAC values. Figure 2 shows the IAC difference diagram.

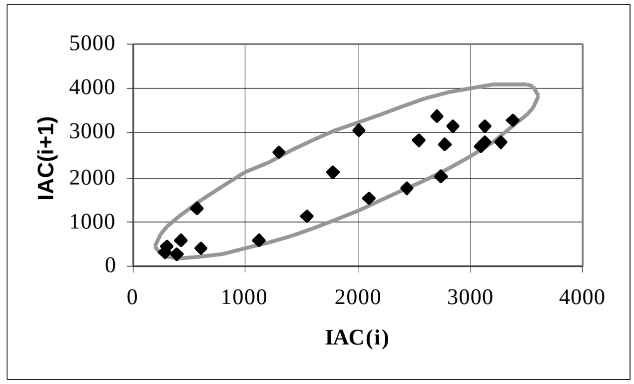

Figure 2: Difference Diagram for IAC Data

This diagram shows that the itinerary of the points in the diagram starts near the origin, rises in a series of values to a maximum, and returns to the origin in another series of lower values. An ellipse, as shown by the gray line in Figure 2, bounds the itinerary of these values. The temporal distribution of IAC values shown in Figure 1 and the itinerary shown in Figure 2 indicate that the IAC values have structure and cannot be described as random numbers with no dependency between pairs of IAC values. Figure 3 shows a parabola (i.e., the gray line) that has been fitted to the rising part of the itinerary of points in the diagram (i.e., the black plotting symbols.)

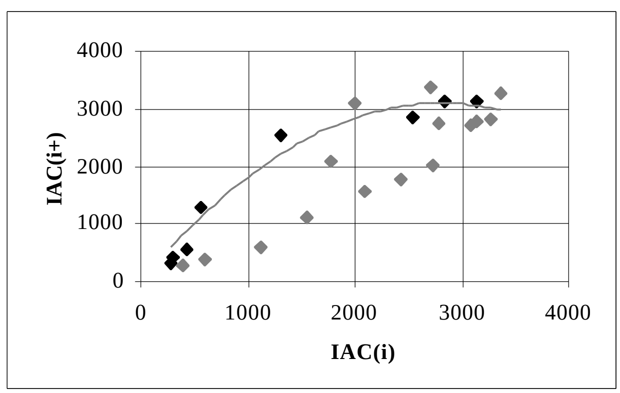

Figure 3: Difference Diagram with Parabola

This parabola corresponds to the logistics equation with $\boldsymbol{R}$ = 2.21 and a goodness of fit parameter of 0.962. The maximum value of the parabola in Figure 3 is 3,099 IACs. The carrying capacity of the airspace can now be estimated by a linear extrapolation of $\boldsymbol{R}$ to the first bifurcation value for the logistics equation; i.e., 3. Hence, the carrying capacity estimate for the airspace in terms of IACs is 4,212.

Summary

Since the air traffic process is dynamic and complex, any model of the air traffic process should reflect the underlying characteristics of the air traffic process such as persistence. The logistics equation does reflect persistence of a population and has been tailored to the air traffic process. Hence, the logistics equation can provide an estimate of the carrying capacity of the air traffic process. This estimate is dependent upon the current air traffic procedures, airline procedures, and operational conditions and may change as these constraints evolve.

References

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Appendix

One way to estimate the change in population of a colony of living creatures is to apply a multiplier to the current value of the population. For example, the change in the population of a colony of cicadas [May 1974] from one generation to the next can be estimated by a simple linear equation:

> The population for the next generation equals some constant times the current population, or in a mathematical format:

$$P_{next} = C * P_{current}.$$

While this equation may, in fact, be adequate to estimate the change in population for two or three generations, this equation cannot be appropriate for any long term forecast since it implies no bounds (e.g., unlimited growth and unbounded food supply) and no problems (e.g., no predators and no diseases.)

A better equation to estimate the change in population of a colony of living creatures acknowledges some of these real-world logistics issues; this equation includes a term proportional to the square of the population and is often called the logistics equation [Chapter 6, Cambel 1993]:

> The population for the next generation equals some constant times the current population minus another constant times the square of the current population, or in a mathematical format:

$$P_{next} = C * P_{current} - D * (P_{current})^2.$$

One reason that the logistics equation provides a much better estimate of the change in population of a colony of living creatures than the first equation is its non-linear characteristics. The non-linear term provides the basis for a more realistic forecast and introduces a surprise in that the logistics equation can have a very complex behavior. Before the advent of digital computers, the complex behavior of the logistics equation was either ignored or assumed to be an artifact of the numerical calculation process [page 9, Cambel 93]. However, the complex behavior of the logistics equation is real and provides additional insight into the population being modeled. As a case in point, the logistics equation provides a basis for estimating the upper bound for the population or carrying capacity for a colony of living creatures.

The logistics equation can be normalized and re-written as:

$$P_{i+1} = \boldsymbol{R} * (P_i - P_i^2)$$

Where "i" is an index for a time interval, P now ranges from 0 to 1, and ***R*** is a constant related to the growth rate.

The solid, black line in Figure A-1 shows the non-linear behavior of the logistics equation in normalized coordinates. In addition, the gray line in Figure A-1 shows the behavior of a linear equation that is tangent to the logistics equation at the origin.

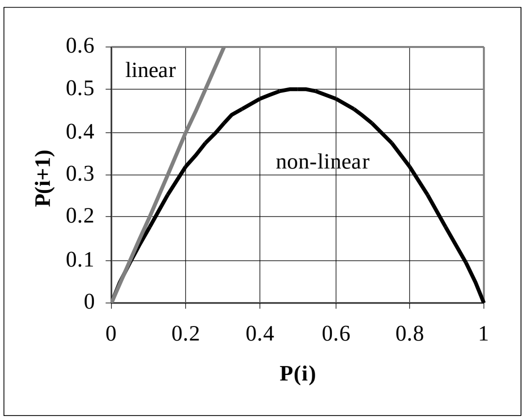

Figure A-1: Normalized Logistics Equation and Linear Approximation

That is, the linear equation is a viable approximation of the non-linear equation for small changes in the population. However, the linear equation is a poor approximation for the entire non-linear equation and the linear equation does not have a upper bound. The complexity of the logistics equation depends on the value of ***R***. For values of ***R*** in range from 0 to 3, the logistics equation has no complexity. However, for values of ***R*** greater than 3, the logistics equation can have more than one value. When ***R*** is greater than 3, the logistics equation has two values; when ***R*** is greater than 3.449, the logistics equation has four values. As ***R*** continues to increase, the logistics equation has more and more values. Hence, the assertion that the logistics equation has complex behavior. Figure A-2 shows the behavior of the logistics equation for a range of ***R*** up to the second bifurcation.

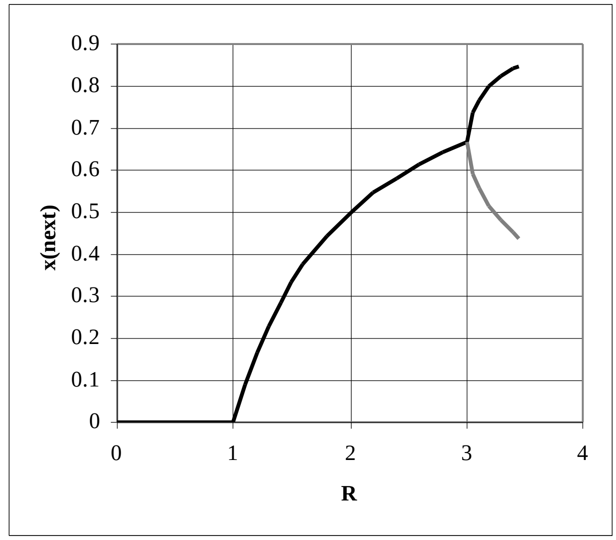

Figure A-2: Bifurcation Diagram and Logistics Equation

In the real world, a value of ***R*** greater than 3 implies that the process is no longer stable and the population can have two or more values [page 54, Cambel 1993]. Hence, the carrying capacity with a stable population occurs when ***R*** equals 3.

The logistics equation "became and remains a useful starting place for the mathematical treatment of population dynamics [page 97, Kingsford 1988]." In fact, the logistics equation has been used in other fields such as the annual US production of copper [Lasky 1951] and the number of articles published in scientific journals [Chapter 1, Price 1963]. In any event, the logistics equation has limitations and three conditions for its use are: "First, it applies to a single variable, and most complex dynamical problems occur exactly because a number of different factors interact. Second, there should be no overlapping of generations. This can be assumed such as in the case of insect species like the gypsy moth, where generations do not overlap. Third, reproduction does not require a pair of parents; only a single parent is necessary [page 122, Cambel 1993]."