A Strategic and Tactical Tool for ATFM Planning based on Statistics and Probability Theory

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ABSTRACT

This paper presents a novel approach based on statistics and probability theory to improve Air Traffic Flow Management (ATFM) efficiency. This tool allows air traffic controllers to organize a more efficient air traffic flow pattern for strategic and tactical planning.

Firstly, a procedure to establish a strategic arrival schedule is developed. An implicit feature of this probabilistic procedure is that it takes into account the uncertainty in the arrival and departure times at a given airport. The methodology has been applied at Glasgow International Airport to design a strategic schedule that reduces either the probability of conflict of the arrivals or the length of the landing slots necessitated by flights. The benefits that are expected through the application of this methodology are a reduction of airborne delays and/or an increase of the airport capacity. Thus a safer and more efficient system is achieved.

Finally, a tactical arrival schedule is presented. This tool is designed to allow air traffic controllers to organise an air traffic flow pattern using a ground holding strategy. During the daily planning of air traffic flow unpredictable events such as adverse weather conditions and system failures occur, necessitating airborne and ground-hold delays. These delays are used by controllers as a means of avoiding 4D-status conflicts. Of the two methods, ground hold delays are preferred because they are safer, less expensive and cause even less pollution. In this paper a method of estimating the duration of a ground-hold for a given flight is developed. The proposed method is novel in the use of a real time stochastic analysis. The method is demonstrated using Glasgow International Airport. The results presented show how a ground hold policy at a departure airport can increase capacity and minimise conflicts at a destination airport.

1. INTRODUCTION

This paper focuses on improving Air Traffic Flow Management (ATFM) efficiency using a statistical and probabilistic approach. ATFM aims to match the demands of aircraft operators to airports and airspace capacity.

Air Traffic Flow Management (ATFM) is [1]:

“ (…) the process that allocates traffic flows to scarce capacity resources (e.g. it meters arrival at capacity constrained airports) (…) Traffic flow management services are designed to meter traffic to taxed capacity resources, both to assure that unsafe levels of traffic congestion do not develop and to distribute the associated movement delays equitably among system users.”

ATFM is a service whose objective is to enable the throughput of the traffic flow to or through areas while guaranteeing a high level of safety. ATFM has the task of allocating resources and facilities of airports and ground structures. Two different types of ATFM services aiming at organising the traffic flows through airports can be distinguished:

Strategic Planning: This service commences seven months before the day under consideration up to twenty-four hours. Schedules and flight plans are computed for the flights due to operate in a given network of airports. A solution that aims at minimising airport congestion while maximising safety and reducing costs is sought.

Tactical Planning: This service is provided during the day of the operations. Its goal is to indicate to air traffic controllers where delays and failures are observed owing to adverse weather conditions, political issues etc. in order that necessary changes to the arrival and departure schedule can be made.

At present [2], the organisations in charge of air traffic flow management are the Central Flow Management Unit

(CFMU) in Europe and the Air Traffic Control System Center (ATCSCC) in the USA. In the USA the absence of national boundaries means that the administration of the traffic flow is more straightforward than in Europe.
The CFMU is mainly involved with tactical planning in order to manage and organise the air traffic situation for en-route purposes (ATFM). The CFMU is not directly involved with planning a strategic schedule, as this issue is mainly a concern for airport management. Tactical planning is usually given two hours before departure according to the expected ATM congestion (both en-route and at the arrival airport). It is then revised according to the actual situation (e.g. adverse weather condition, ATC centre overloaded). Flights are only updated in the CFMU systems when they take-off or when they enter a country through an AFTN message issued by ATC. Currently, further developments are taking place in the collection of actual flight information. The intention is that this will then be distributed to airports and ATC centres in order to optimise ATFM at short notice.

As stated in [3], the problem of assigning to aircraft the necessary ground hold time so that total delay cost is minimised will here after be referred to as the ground holding problem (GHP). Two different versions of GHP can be distinguished according to the moment in which the decisions are taken: *static* and *dynamic*. In the first case, the ground and airborne holds are established once at the beginning of the day. In the second case the amount of ground hold and airborne delay is assigned during the course of the day taking into account new information available real time. The parameters involved in the GHP can be modelled either as *deterministic* or as *random* variables giving respectively a deterministic or a probabilistic version of GHP.

There has been much research concerning GHP, however [4] is the first author to have given a methodical and a regular description of the GHP problem. [5] proposed a dynamic programming algorithm to solve the single airport static probabilistic GHP once the probability density function (PDF) of the capacity of the airport involved is known. The first systematic attempt to examine a network of airports is given by [6], where a static deterministic multi airport GHP is analysed. In [7] a multi airport probabilistic formulation for the static and dynamic GHP is given. In this model different possible scenarios of airport capacities are introduced giving a probabilistic GHP.

This paper is novel as it presents and introduces a new approach to ATFM at both the strategic and tactical planning that aims:

- either to reduce the probability of conflict between aircraft arriving at a given airport, which will decrease the likelihood of airborne delay. Thus the benefits expected by the implementation of this approach are a reduction of the costs for the airlines, an improvement of the safety of airports operations and even a reduce pollution.
- or to reduce the length of the slots required by flights, which will increase the airport capacity[†].
- or a combination of these two options.

The core of this new approach is to model arrival time of aircraft as real random variables in order to solve:

- strategic planning.
- tactical planning (single airport dynamic probabilistic GHP).

Owing to unpredictable events, changes may be required to the planned schedule at very short notice. As shown, the daily schedule planning is called tactical planning. Presuming the availability of a data link support between aircraft and airport a tactical planning methodology, based on a dynamic probabilistic GHP, is shown in this paper. The methodology proposes that the aircraft, throughout its flight, relays its current position to the arrival airport. This data will then be computed real-time by the system to propose a new tactical planning. Ground hold and airborne delay will be then allocated and finally communicated to aircraft and airports.

2. MATHEMATICAL MODEL

2.1. Nomenclature

$\overset{\Delta}{=}$	is defined to be
$\boldsymbol{R}$	set of real numbers
$\boldsymbol{T}$	set of the points belonging to a time period
$\boldsymbol{F}$	set of flights
A	departure airport
B	arrival airport
P	generic point
$d(\cdot,\cdot)$	euclidean distance
Ω	experiment
ζ	outcome of the experiment
t_s^a	**S**cheduled **A**rrival **T**ime
t_s^d	**S**cheduled **D**eparture **T**ime
$\mathbf{d^a}$	**A**rrival **D**elay
$\mathbf{t^a}$	**A**rrival **T**ime
$\mathbf{t^t}$	**T**ravelling **T**ime
$\underline{\mathbf{v}}$	**V**elocity
$\mathcal{A}$	set in which the arrival delay is defined
$\mathcal{B}$	set in which the arrival time is defined
$\mathcal{C}$	set in which the departure time is defined
$\mathcal{E}$	set in which the travelling time is defined
$\mathcal{F}$	set in which the velocity is defined

[†] For the definition and further details of airport capacity see [8] and [9].

2.2. Arrival Time as a Real Random Variable

According to [10], a random variable "… consists of an experiment **Ω** with a probability measure P[.] defined on sample space S and a function that assigns a real number to each outcome in the sample space of the experiment **Ω**".

In this section the arrival time of an aircraft is defined as a real random variable:

$$\mathbf{t^a} \stackrel{\Delta}{=} t_s^a + \mathbf{d^a} \qquad \begin{cases} \mathbf{t^a} \subseteq \mathcal{B} \\ t_s^a \in \boldsymbol{R} \\ \mathbf{d^a} \subseteq \mathcal{A} \end{cases}$$

where :

$t_s^a \stackrel{\Delta}{=}$ **S**cheduled **A**rrival **T**ime of a given flight[†]. This is a deterministic variable having as domain the set of the real number ***R***.

$\mathbf{d^a} \stackrel{\Delta}{=}$ **A**rrival **D**elay for a given flight. This is modelled as a real random variable defined in the set $\mathcal{A}$. This choice has been made due to the intrinsic unpredictability affecting delays. For an aircraft operating a given flight, an arrival delay (positive or negative) can be observed. To each outcome $\zeta \in \mathcal{A}$ of the experiment **Ω**, a real number $\mathbf{d^a}(\zeta) \in \boldsymbol{R}$ is assigned.

$\mathbf{t^a} \stackrel{\Delta}{=}$ **A**rrival **T**ime for a selected flight. This variable is the sum of a deterministic and of a real random variable. The result gives a real random variable defined in the set $\mathcal{B}$.

For clarity, all random variable will be written in boldface letters $\mathbf{t^a}$, $\mathbf{d^a}$. All sets, in which random variable are defined, will be typefaced with the following style $\mathcal{A}$, $\mathcal{B}$.

2.3. Building Probability Density Function (PDF)

Real random variables have been introduced to model the ATFM issue. To build the Probability Density Functions (PDFs) of these random variables, real data provided by the Civil Aviation Authority CAA are used. The data provide the real and the scheduled arrival times for every flight landing in Glasgow International Airport between 1st April 1998 and 30th of September 1998.

The experiment **Ω** is defined in the following step:

ζ = difference between actual landing time and scheduled arrival time

[†] Throughout this paper the word *flight* refers to a regular scheduled service between the departure and arrival airport.

The random variable $\mathbf{d^a}$ is defined by:

$$\mathbf{d^a}(\zeta) \stackrel{\Delta}{=} \zeta$$

The PDF of $\mathbf{d^a} \subseteq \mathcal{A}$ is computed as histogram of the relative frequency of occurrences. This histogram is considered equivalent to the PDF by applying the *Law of large numbers* [11]:

If the experiment Ω is repeated n times and the event a occurs n_a times, then, with a high degree of certainty, the relative frequency n_a/n of the occurrence of a is close to p=Pr(a),

$$\Pr(a) \cong \frac{n_a}{n}$$

provided n sufficiently large.

Suppose this experiment **Ω** is performed *n* times, at a given outcome ζ of the experiment the real random variable $\mathbf{d^a} \subseteq \mathcal{A}$ associates a value $\mathbf{d^a}(\zeta) \in \boldsymbol{R}$. In conclusion, the Probability Density Function (PDF) *f(x)* for a given *x* is computed as ratio between the number of trials such as $x \le \mathbf{d^a}(\zeta) \le x+\Delta x$ and the total number of trials *n*.

$$f(x)\Delta x \cong \frac{\Delta n(x)}{n}$$

Provided *n* sufficiently large and Δ*x* sufficiently small.

2.4. Strategic planning

Let $(t_{s1}^d, t_{s2,\dots,}^d t_{sn}^d) \in \boldsymbol{R}^n$ be the departure scheduled time of a set of flights $\{1,2,\dots n\} \in \boldsymbol{F}$. Let $\{\mathbf{t_1^a}, \mathbf{t_2^a}, \dots \mathbf{t_n^a}\} \subseteq \mathcal{B}^n$ be the arrival times of ***F***.

The task is to compute the shifting times $(T_{12,} T_{23,} \dots T_{(n-1)n}) \in \boldsymbol{R}^{n-1}$ to satisfy the conditions

$$\begin{cases} \Pr\{\mathbf{t_2^a} + T_{12} > \mathbf{t_1^a}\} = \alpha_{12} \\ \Pr\{\mathbf{t_3^a} + T_{23} > \mathbf{t_2^a}\} = \alpha_{23} \\ \vdots \\ \Pr\{\mathbf{t_n^a} + T_{(n-1)n} > \mathbf{t_{n-1}^a}\} = \alpha_{(n-1)n} \end{cases}$$

where T_{12}, T_{23}, … $T_{(n-1)n} \in \boldsymbol{R}^{n-1}$ are the required unknowns representing the time to shift the random variable $\mathbf{t_2^a}, \dots \mathbf{t_n^a} \subseteq \mathcal{B}^n$. The term $\Pr\{\mathbf{t_i^a} + T_{ij} > \mathbf{t_j^a}\}$ expresses the probability one random variable is greater than the other one.

To compute the unknowns $T_{12}, T_{23}, \dots T_{(n-1)n} \in \boldsymbol{R}^{n-1}$, this probability is imposed by fixing the terms:

$$\alpha_{12,}, \alpha_{23}, \dots \alpha_{(n-1)n} \in [0,1]^{n-1}$$

Denoting by $\mathbf{z_{ij}} = \mathbf{t_i^a} - \mathbf{t_j^a}$, the following equivalence is derived:

$$\Pr\{\mathbf{t_i^a} > \mathbf{t_j^a}\} \quad \Leftrightarrow \quad \Pr\{\mathbf{z_{ij}} > 0\}$$

With the previous position the difference between two real random variables $\mathbf{t_i^a}, \mathbf{t_j^a}$ is leaded in the study of only one real random variable $\mathbf{z_{ij}}$.

$$f_{\mathbf{z_{ij}}}(z_{ij}) = \int_{-\infty}^{+\infty} f_{\mathbf{t_i^a t_j^a}}(z_{ij} + t_j)dt_j$$

where $f_{\mathbf{t_i^a t_j^a}}\left(t_i^a, t_j^a\right)$ is the joint PDF of the real random variables $\mathbf{t_i^a} \subseteq \mathcal{B}$ and $\mathbf{t_j^a} \subseteq \mathcal{B}$.

Suppose that $\mathbf{t_i^a} \subseteq \mathcal{B}$ and $\mathbf{t_j^a} \subseteq \mathcal{B}$ are independent real random variables.

$$f_{z_{ij}}(z_{ij}) = \int_{-\infty}^{+\infty} f_{t_i^a}(z_{ij} + t_j) f_{t_j^a}(t_j)dt_j = \int_{-\infty}^{+\infty} f_{t_i^a}(t_i) f_{t_j^a}(t_i - z_{ij})\, dt_i$$

This is an important result as the integral shown above is very similar to the well-known integral of convolution.

Solving the following system formed by *n-1* separated equations with the *n-1* unknowns $T_{12}, T_{23}, \ldots T_{(n-1)n} \in \boldsymbol{R}^{n-1}$, the required shifting times are calculated.

$$\begin{cases} \int_{-\infty}^{T_{12}} \int_{-\infty}^{+\infty} f_{\mathbf{t_1^a}}(z_{12} + t_2) f_{\mathbf{t_2^a}}(t_2)dt_2 dz_{12} = \alpha_{12} \\ \int_{-\infty}^{T_{23}} \int_{-\infty}^{+\infty} f_{\mathbf{t_2^a}}(z_{23} + t_3) f_{\mathbf{t_3^a}}(t_3)dt_3 dz_{23} = \alpha_{23} \\ \vdots \\ \int_{-\infty}^{T_{(n-1)n}} \int_{-\infty}^{+\infty} f_{\mathbf{t_{n-1}^a}}(z_{(n-1)n} + t_n) f_{\mathbf{t_n^a}}(t_n)dt_n dz_{(n-1)n} = \alpha_{(n-1)n} \end{cases}$$

Delays can be presented as stochastic processes as they are time dependent. The aircraft scheduled to land at $t_s^a \in \boldsymbol{R}$ has a delay $\mathbf{d^a} \subseteq \mathcal{A}$, this scheduled time can be changed into a new scheduled time $t_s^{a'} \in \boldsymbol{R}$. Thus, in general another delay $\mathbf{d^{a'}} \subseteq \mathcal{A}$ is defined.

The task of this section is to present a methodology for strategic scheduling and thus scheduled times $t_s^a \in \boldsymbol{R}$ will be affected by changes. These adjustments of the scheduled times will be small when they are compared with the whole interval time required by PDFs. Hence, delays are hypothesised stationary random variables:

$$\mathbf{d^a}\left(t_s^a\right) = \mathbf{d^a}\left(t_s^a + T\right) \quad \begin{cases} \forall T \in \boldsymbol{R} \\ \mathbf{d^a} \subseteq \mathcal{A} \\ t_s^a \in \boldsymbol{R} \end{cases}$$

With this hypothesis, the same delays $\mathbf{d^a} \subseteq \mathcal{A}$ are observed even if the departure times $t_s^a \in \boldsymbol{R}$ are changed.

The old and new departure schedule is shown in the Table 1 while the old and the new arrival schedule is shown in Table 2.

	Old departing scheduling	**New departing scheduling**
Aircraft 1	t_{s1}^d	t_{s1}^d
Aircraft 2	t_{s2}^d	$t_{s2}^d + T_{12}$
…	…	…
Aircraft n	t_{sn}^d	$t_{sn}^d + \sum_{k=2}^{n} T_{(k-1)k}$

Table 1: Old and new departure schedule

	Old arrival scheduling	**New arrival scheduling**
Aircraft 1	t_{s1}^a	t_{s1}^a
Aircraft 2	t_{s2}^a	$t_{s2}^a + T_{12}$
…	…	…
Aircraft n	t_{sn}^a	$t_{sn}^a + \sum_{k=2}^{n} T_{(k-1)k}$

Table 2: Old and new arrival scheduling

2.5. Travelling Time as a Real Random Variable, Velocity as a Two Dimensional Real Random Variable

The real random variable travelling time is defined as difference between the arrival time and the scheduled departure time.

$$\mathbf{t^t} \stackrel{\Delta}{=} \mathbf{t^a} - t_s^d \qquad \begin{cases} \mathbf{t^t} \subseteq \mathcal{E} \\ \mathbf{t^a} \subseteq \mathcal{B} \\ t_s^d \in \boldsymbol{R} \end{cases}$$

$t_s^d \stackrel{\Delta}{=}$ **S**cheduled **D**eparture **T**ime of a given flight. This is a deterministic variable having as domain the set of the real number ***R***.

$\mathbf{t^t} \overset{\Delta}{=}$ **T**ravelling **T**ime of a given flight. The differences between the arrival and departure times of aircraft operating a given flight are observed. A real random variable is defined assigning a real number to each of these observations. $\mathcal{E}$ is called the set where this random variable is defined. To every outcome $\zeta \in \mathcal{E}$ of the experiment **Ω,** a real number $\mathbf{t^t}(\zeta) \in \boldsymbol{R}$ is assigned.

A set of Cartesian axes is defined as follows:

- the axis τ, whose unit vector is labeled with $\underline{\tau}$, is along-track direction of the aircraft
- the axis n, whose unit vector is labeled with $\underline{\mathrm{n}}$, is cross-track direction of the first aircraft

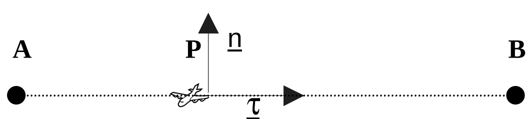


Figure 1: Cartesian axes

Velocity is defined as a two dimensional real random variable.

$$\underline{\mathbf{v}} \overset{\Delta}{=} (\mathbf{v_\tau}, \mathbf{v_n}) \qquad \underline{\mathbf{v}} \subseteq \mathcal{F}$$

$\underline{\mathbf{v}} \overset{\Delta}{=}$ **V**elocity of the aircraft. This is a two dimensional random variable. The measurements of the velocities in fixed instant of times for given aircraft operating a selected flight are observed. A two dimensional real random variable is defined assigning a pair of real numbers to each of these observations. $\mathcal{F}$ is called the set where this random variable is defined. To every outcome $\zeta \in \mathcal{F}$, a pair of real number are assigned $v_\tau(\zeta) \in \boldsymbol{R}$, $v_n(\zeta) \in \boldsymbol{R}$.

For clarity, all multidimensional random variables will be written outlined in boldface letters $\underline{\mathbf{v}}$.

2.6. Velocity as stochastic process

The random variable velocity is defined by the experiment **Ω**. The outcome ζ, of the experiment **Ω** is here given:

ζ = measurement of aircraft velocity in a fixed instant of time during a selected flight for a given aircraft.

A two dimensional real random variable $\underline{\mathbf{v}}(\zeta) \subseteq \mathcal{F}$ is defined for a given instant of time. If $t_1, t_2, t_3, \ldots t_n$ $(t_1, t_2, t_3, \ldots t_n) \in \boldsymbol{T^n}$ are instants of time at which the experiment **Ω** is performed, the outcome of **Ω** will be the set of values $\{\underline{\mathbf{v}}(\zeta_1, t_1), \underline{\mathbf{v}}(\zeta_2, t_2), \underline{\mathbf{v}}(\zeta_3, t_3), \ldots \underline{\mathbf{v}}(\zeta_n, t_n)\}$. Therefore, a function of two variables t and ζ is defined. For a fixed t, the two dimensional random variable speed $\underline{\mathbf{v}}(\zeta) \subseteq \mathcal{F}$ is obtained. This indicates the probability of occurrence of a certain speed. For a specific ζ, the time function v(t) indicating the time-record of the speed during the flight is obtained. Hence, the family of random variables $\underline{\mathbf{v}}(\zeta, t) \subseteq \mathcal{F}x\boldsymbol{T}$ is defined to be a stochastic process.

2.7. Velocity as a stationary stochastic process

An aircraft which goes from the point $\mathbf{A}(A_x, A_y) \in \boldsymbol{R^2}$ to the point $\mathbf{B}(B_x, B_y) \in \boldsymbol{R^2}$ is given. Let $\underline{\mathbf{v}}(\zeta;t) \subseteq \mathcal{F}x\boldsymbol{T}$ be the stochastic process velocity of this aircraft. The marginal PDFs of the two dimensional random variable velocity $\underline{\mathbf{v}}(\zeta;t) \subseteq \mathcal{F}x\boldsymbol{T}$ along and across track are defined as:

$$\Pr\begin{Bmatrix} \text{aircraft} \\ \text{has velocity} \\ v_\tau(t) \end{Bmatrix} = f_{v_\tau}(v_\tau;t) = \int_{-\infty}^{+\infty} f_{\underline{v}(v_\tau, v_n)}(v_\tau, v_n; t)\,dv_n$$

$$\Pr\begin{Bmatrix} \text{aircraft} \\ \text{has velocity} \\ v_n(t) \end{Bmatrix} = f_{v_n}(v_n;t) = \int_{-\infty}^{+\infty} f_{\underline{v}(v_\tau, v_n)}(v_\tau, v_n; t)\,dv_\tau$$

Without considering taking off and landing time (where speed is affected by substantial changes), the hypothesis that the speed is time dependent is not necessary. For a given instant of time during aircraft cruise, a velocity distribution indicating the probability of occurrences of a given speed can be computed. Therefore, the stochastic process velocity $\underline{\mathbf{v}}(\zeta, t) \subseteq \mathcal{F}x\boldsymbol{T}$ is assumed to be stationary.

$$\underline{\mathbf{v}}(\zeta, t) = \underline{\mathbf{v}}(\zeta) \quad \Leftrightarrow \quad \begin{cases} \mathbf{v}_\tau(\zeta; t) = \mathbf{v}_\tau(\zeta) \\ \mathbf{v_n}(\zeta; t) = \mathbf{v_n}(\zeta) \end{cases} \quad \forall t \in \boldsymbol{T}$$

Hypothesising the stochastic process $\underline{\mathbf{v}}(\zeta;t) \subseteq \mathcal{F}x\boldsymbol{T}$ be stationary $\underline{\mathbf{v}}(\zeta) \subseteq \mathcal{F}$, the PDFs, (each of them being calculated for different instant times) are assumed to be identical, see Figure 2.

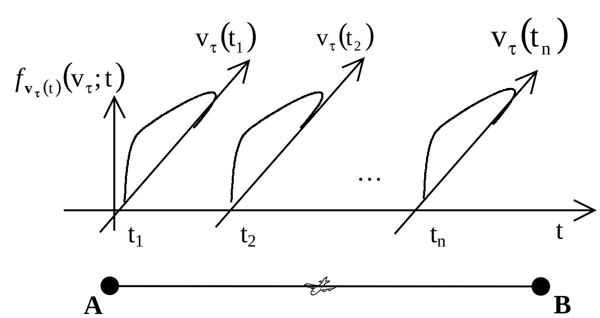


Figure 2: PDF of the along track velocity

2.8. Considerations to build the PDF of the along track velocity

Due to the lack of data regarding the experiments defining the random variable velocity $\underline{\mathbf{v}}(\zeta)\subseteq\mathcal{F}$, the PDF of the velocity along the track is obtained through a transformation of variables from related experiments from which data are available. Thus, considering the work described [12], the PDF of the random variable travelling time will be transformed into the PDF of the velocity along the track assuming certain simplifying hypothesis. The PDF of the travelling time is obtained using the data provided by the CAA.

The departure and arrival airports are supposed to be placed in **A** and **B**. For a given journey, a travelling time $t^{t}_{AB}(\zeta)\in\boldsymbol{R}$ is observed. The temporal mean of the velocity $\overline{v_\tau}(\zeta)$ is computed through the following equation (see Figure 3):

$$\overline{v_\tau}(\zeta)=\frac{d(\mathbf{A},\mathbf{B})}{t^{t}_{AB}(\zeta)} \qquad \begin{cases} d(\mathbf{A},\mathbf{B})\in\boldsymbol{R} \\ t^{t}_{AB}(\zeta)\in\boldsymbol{R} \\ \overline{v_\tau}(\zeta)\in\boldsymbol{R} \end{cases}$$

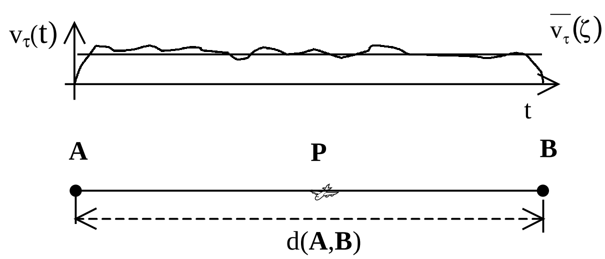


Figure 3: For a given journey $\overline{v_\tau}(\zeta)$ is observed

Observing this temporal mean *n* times a random variable is built that represents the distribution of the mean of the velocity.

$$\overline{\mathbf{v}_\tau}=g\left(\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}\right)=\frac{d(\mathbf{A},\mathbf{B})}{\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}} \qquad \begin{cases} d(\mathbf{A},\mathbf{B})\in\boldsymbol{R} \\ \mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}\subseteq\mathcal{E} \\ \overline{\mathbf{v}_\tau}\subseteq\mathcal{F} \end{cases}$$

Supposing that $\overline{\mathbf{v}_\tau}=\mathbf{v}_\tau$, then $\mathbf{v}_\tau$ is a real random variable defined not directly for each experimental outcome. Alternatively $\mathbf{v}_\tau$ is defined indirectly via the real random variable $\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}\subseteq\mathcal{E}$ and the function g($\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}$).

2.9. Applied Tactical Planning based on a dynamic probabilistic GHP

The velocity along the track $\mathbf{v}_\tau(\zeta)\subseteq\mathcal{F}$ is computed through the ratio between the travelling time $\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}\subseteq\mathcal{E}$ of the route **AB** and the euclidean distance d(**A**,**B**)$\in\boldsymbol{R}$ between the point **A** and the point **B**

$$\mathbf{v}_\tau=\frac{d(\mathbf{A},\mathbf{B})}{\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}} \qquad \begin{cases} \mathbf{v}_\tau\subseteq\mathcal{F} \\ d(\mathbf{A},\mathbf{B})\in\boldsymbol{R} \\ \mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}\subseteq\mathcal{E} \end{cases}$$

The travelling time $\mathbf{t}^{\mathbf{t}}_{\mathbf{PB}}\subseteq\mathcal{E}$ of the aircraft for the route **PB** is:

$$\mathbf{t}^{\mathbf{t}}_{\mathbf{PB}}=\frac{d(\mathbf{P},\mathbf{B})}{\mathbf{v}_\tau} \qquad \begin{cases} d(\mathbf{P},\mathbf{B})\in\boldsymbol{R} \\ \mathbf{v}_\tau\subseteq\mathcal{F} \\ \mathbf{t}^{\mathbf{t}}_{\mathbf{PB}}\subseteq\mathcal{E} \end{cases}$$

Substituting the velocity $\mathbf{v}_\tau(\zeta)\subseteq\mathcal{F}$, the travelling time $\mathbf{t}^{\mathbf{t}}_{\mathbf{PB}}\subseteq\mathcal{E}$ of the route **PB** can be expressed as travelling time $\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}\subseteq\mathcal{E}$ of the route **AB**

$$\mathbf{t}^{\mathbf{t}}_{\mathbf{PB}}=\frac{d(\mathbf{P},\mathbf{B})}{d(\mathbf{A},\mathbf{B})}\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}} \qquad \begin{cases} d(\mathbf{P},\mathbf{B})\in\boldsymbol{R},\ d(\mathbf{A},\mathbf{B})\in\boldsymbol{R} \\ \mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}\subseteq\mathcal{E} \\ \mathbf{t}^{\mathbf{t}}_{\mathbf{PB}}\subseteq\mathcal{E} \end{cases}$$

The arrival time $\mathbf{t}^{\mathbf{a}}_{\mathbf{PB}}\subseteq\mathcal{B}$ in **B** once the aircraft has departed from **P** is:

$$\mathbf{t}^{\mathbf{a}}_{\mathbf{PB}}=\mathbf{t}^{\mathbf{d}}_{\mathbf{PB}}+\mathbf{t}^{\mathbf{t}}_{\mathbf{PB}}=$$

$$=\mathbf{t}^{\mathbf{d}}_{\mathbf{PB}}+\frac{d(\mathbf{P},\mathbf{B})}{d(\mathbf{A},\mathbf{B})}\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}} \qquad \begin{cases} d(\mathbf{P},\mathbf{B})\in\boldsymbol{R} \\ d(\mathbf{A},\mathbf{B})\in\boldsymbol{R} \\ \mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}\subseteq\mathcal{E} \\ \mathbf{t}^{\mathbf{d}}_{\mathbf{PB}}\subseteq\mathcal{A} \end{cases}$$

If the departure time from **P** is known the random variable $\mathbf{t}^{\mathbf{d}}_{\mathbf{PB}}\subseteq\mathcal{A}$ simplifies in a mere scalar $t^{d}_{PB}(\zeta)\in\boldsymbol{R}$. In a real implementation of this methodology, the departure time from **P** can be known though a data-link message between aircraft and ground.

$$\mathbf{t}^{\mathbf{a}}_{\mathbf{PB}} = t^{d}_{PB}(\zeta) + \frac{d(\mathbf{P},\mathbf{B})}{d(\mathbf{A},\mathbf{B})}\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}} \quad \begin{cases} \mathbf{t}^{\mathbf{a}}_{\mathbf{PB}} \subseteq \mathcal{B} \\ t^{d}_{PB} \in \boldsymbol{R} \\ d(\mathbf{P},\mathbf{B}) \in \boldsymbol{R} \\ d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^{\mathbf{t}}_{\mathbf{AB}} \subseteq \mathcal{E} \end{cases}$$

This formula solves the arrival time in the destination airport **B** once is know the departure time of the aircraft in **P**. Note the departure time from **P** is equal to the arrival time in **P**.
The parameters involved are the eucledian distances d(**P**,**B**)∈***R***, d(**A**,**B**)∈***R***, the real random variable travelling time $\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}} \subseteq \mathcal{E}$ and the departure time from **P** $t^{d}_{PB} \in \boldsymbol{R}$.

Applying the *Fundamental Theorem of transformation of variables* [10], the PDF of the arrival time in the destination airport **B** $\mathbf{t}^{\mathbf{a}}_{\mathbf{PB}} \subseteq \mathcal{B}$ is derived by:

$$f_{\mathbf{t}^{\mathbf{a}}_{\mathbf{PB}}}\left(t^{a}_{PB}\right) = \frac{|d(\mathbf{A},\mathbf{B})|}{|d(\mathbf{P},\mathbf{B})|} f_{\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}}}\left(\left(t^{t}_{PB} - t^{d}_{PB}\right)\frac{d(\mathbf{A},\mathbf{B})}{d(\mathbf{P},\mathbf{B})}\right)$$

In conclusion, the PDF $f_{\mathbf{t}^{\mathbf{a}}_{\mathbf{PB}}}\left(t^{a}_{PB}\right)$ of the arrival time in **B** is given using only:

- the PDF of the travelling time of the route **AB** $\mathbf{t}^{\mathbf{t}}_{\mathbf{AB}} \subseteq \mathcal{E}$ which is known through real-data.
- the departure time from **P**, $t^{d}_{PB} \in \boldsymbol{R}$ which is supposed to be known through a data-link message.

3. RESULTS

3.1. Strategic planning

Probability Density Functions (PDFs) of the delay are built using real data covering the period commencing 1st April 1998 up to 30th September 1998. There follows an analysis of the British Airways flight from Birmingham (UK), scheduled to arrive at Glasgow International Airport (UK) at 8:25 a.m.. In Figure 4 samples of the delays $\mathbf{d}^{\mathbf{a}}(\zeta_1), \mathbf{d}^{\mathbf{a}}(\zeta_2), \ldots \mathbf{d}^{\mathbf{a}}(\zeta_{130}) \in \boldsymbol{R}^{130}$ are shown. These delays are the difference between the real landing time and the scheduled arrival time, observed over 130 consecutive days, using real data.

The origin of the PDF is set at the scheduled arrival time, for the studied flight at 8:25 a.m. The points placed before zero show flights coming earlier than planned while the measurements positioned after the origin show delays. The vertical axis gives the probability that a fixed delay occurred. Given the fact that this is a short haul flight, arrival times (PDF) are very close each other.

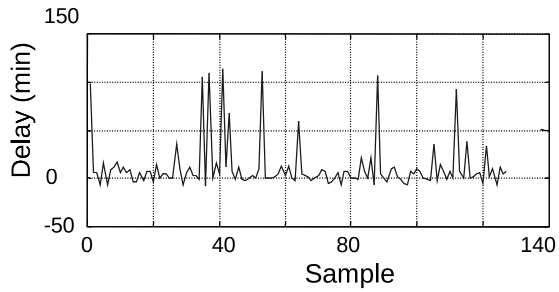


Figure 4: Samples of the delays for British Airways

In Figure 5 the PDF $f_{\mathbf{d}^{\mathbf{a}}}\left(d^{a}\right)$ is displayed.

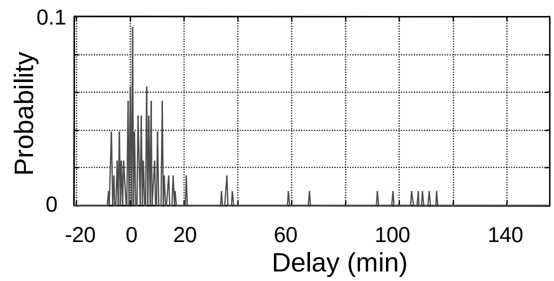


Figure 5: PDF of the delays of British Airways

In Figure 6 the PDF of Air Canada flight flying from Toronto (CA) to Glasgow (UK) scheduled to arrive at 8:00 a.m. is displayed. Owing to the length of the journey, the arrival times are effected by a higher uncertainty.

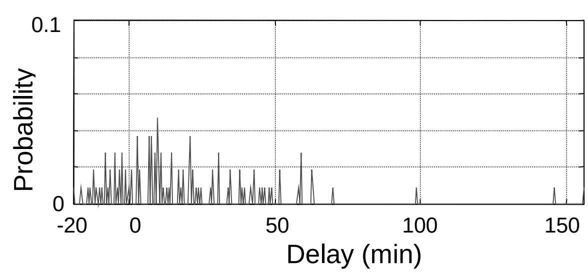


Figure 6: PDF of the delays of Air Canada

Studying PDFs of different flights arriving at Glasgow International Airport, long distant flights show that are effected by a greater uncertainty than short distance flights. This is caused by variable factors (e.g. wind, Air Traffic Centre overloaded etc.) encountered on the route. Using Glasgow International Airport as an example, the time interval between 8:00 a.m. until 8:25 a.m. is anlaysed. As shown in Table 3, four aircraft are scheduled to land. This interval is chosen both because it is one of the busiest period during the day and because a long journey flight is involved. In Table 3, the flights and their scheduled arrival time $t^{a}_{s1}, t^{a}_{s2}, t^{a}_{s3}, t^{a}_{s4} \in \boldsymbol{R}^{4}$ are listed.

nº	Flight	Label	Departure	Scheduled Arrival Time (a.m.)
1	Aer Lingus	AE	Dublin	t^a_{s1} =8:00
2	Air Canada	AC	Toronto	t^a_{s2} =8:00
3	British Airways	BA	Birmingham	t^a_{s3} =8:25
4	European Air Charter	EAC	Bournemouth	t^a_{s4} =8:25

Table 3: Current scheduling

In Figure 7 the total distribution of the arrival time is computed. For a fixed instant time in time, this PDF gives the probability that at least one of the four aircraft is coming. Two main peaks are distinguishable, indicating two intervals time with high probability of arrivals.

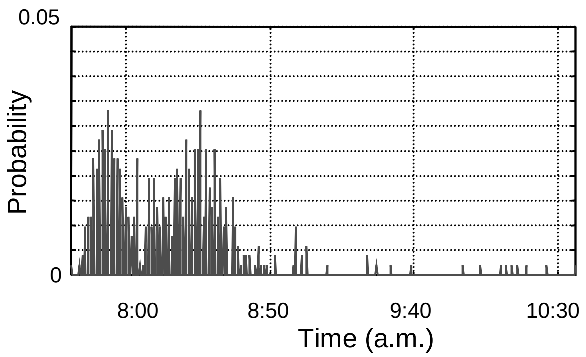


Figure 7: Total distribution of the arrivals according to the current scheduling

Although from the scheduling an interval commencing at 8:00 a.m. until 8:25 a.m. is considered, Figure 7 shows aircraft landing even later, so a longer time period must be considered. Define Arrival Slot (AS) as "*The shortest time period in which 90% of the flights arrive.*". As shown in Figure 8 in the current scheduling AS=56 is minutes long. According to the current aeronautical legislation, the minimum aircraft time separation in the landing procedure is 3 minutes. In order to guarantee such a temporal distance during peak hours, operators allocate airborne and ground hold delays. Probability of Conflict (PC) is defined as "*The probability of at least two aircraft arriving within a time difference of 3 minutes*". The higher the probability of conflict, the higher the likelihood of airborne delay having to be allocated, since if a conflict is deemed to occur, the air traffic controllers would have to issue separation instructions to the aircraft in order to achieve the adequate time spacing at the runway. Thus, the probability of conflict should be reduced to increase efficiency, reduce cost and improve the levels of safety. As shown in Figure 8, PC in the current scheduling is equal to 35.4%.

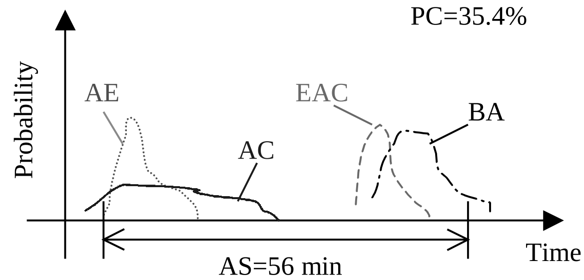


Figure 8: Current situation

In Table 4 two new strategic schedules are proposed giving the option of:

- decrease of airborne delay by 15.3% reducing the Probability of Conflict (PC) by the same quantity. See Figure 9.
- improve the airport capacity by 14.3% reducing the Arrival Slot (AS) of the same quantity. See Figure 10.
- a combination of the two options above described.

Flight	NEW STRATEGIC SCHEDULE	
	Fixed AS=56min PC=30.0% Reduction by 15.3%	**Fixed PC=35.4% AS=48min** Reduction by 14.3%
AE	8:00 a.m.	8:00 a.m.
EAC	8:07 a.m.	8:09 a.m.
AC	8:28 a.m.	8:12 a.m.
BA	8:33 a.m.	8:18 a.m.

Table 4: Proposed new scheduling

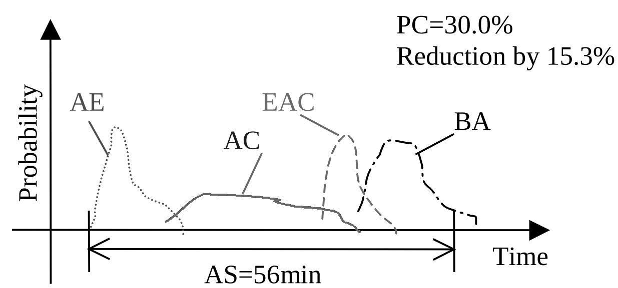


Figure 9: Strategic Planning: fixed AS

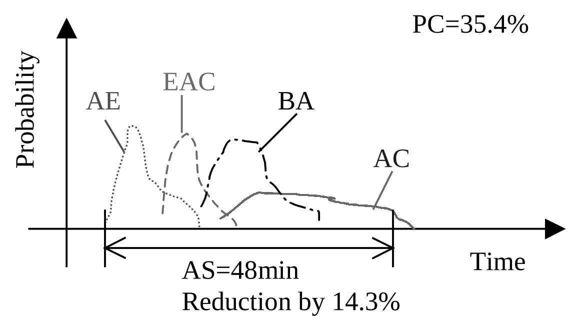


Figure 10: Strategic Planning: fixed PC

3.2. Tactical Planning based on a dynamic probabilistic GHP

When the Air Canada (AC) flight arrives at a point located **P=**1000 Km away from Glasgow International Airport, the aircraft transmits the time $t_{PB}^{d}(\zeta) \in \boldsymbol{R}$ to the arrival airport placed in **B**. The traffic manager codes this data into the computer updating the PDF of the delay and thus the PDF of the arrival time.

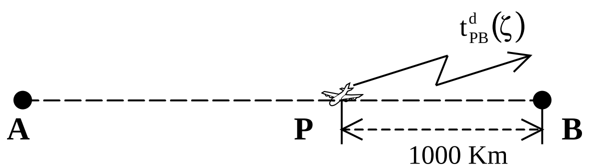


Figure 11: Air Canada transmits its position

If the flight arrives early at the point placed **P=**1000 Km away from the airport, it will be allocated as first in the arrival order while if it is late it will be placed as the last one. In all the intermediate situations, depending of the amount of the delay, it will be allocated between the flights.

Through the tactical planning, the airport managers can have control over flights using data link capabilities. Suggestions to pilots to increase, decrease or keep the same velocity can be given. To reduce complexity, the example to be considered will not use the feed back capability. To study the affect of the data link, it is enough to run the program again simulating the new data.

Updating the information, the PDFs of the arrival time for given flights seem compressed, as shown in Figure 12. The uncertainty is clearly reduced so there is more accuracy in predicting the arrival time. The closer the aircraft is to the airport minor uncertainty effects ETA. In Figure 11 the PDFs for the Air Canada flight when recalculated using the updated information from a point **P**=1000 Km away from Glasgow International Airport is shown.

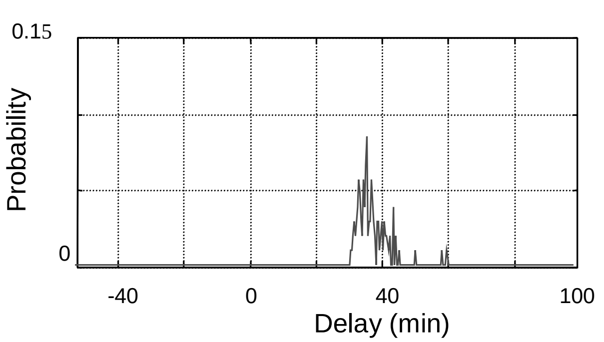


Figure 12: Transformed PDF of delays for Air Canada

The task now is how to allocate the landing of Air Canada flight between the others when the time of reaching the point **P=**1000 Km away from Glasgow International Airport is known. Four cases are analysed as shown in Figure 13:

1. The aircraft comes very early and lands first, before the Aer Lingus (AE) flight.
2. The aircraft is nearly on time. To optimise efficiency it is scheduled to land in the second position after the Aer Lingus (AE) flight.
3. The aircraft is nearly on time. To optimise efficiency it is scheduled to land in the third position after the European Air Charter (EAC) flight.
4. The aircraft is delayed. It lands last after the British Airways (BA) flight.

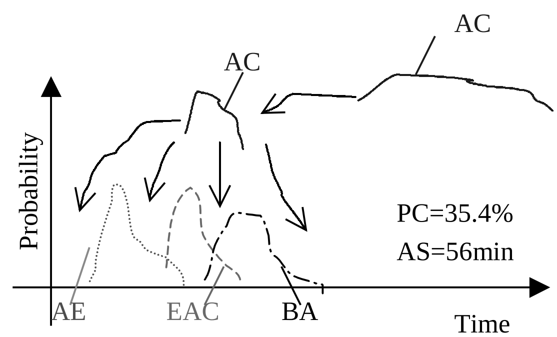
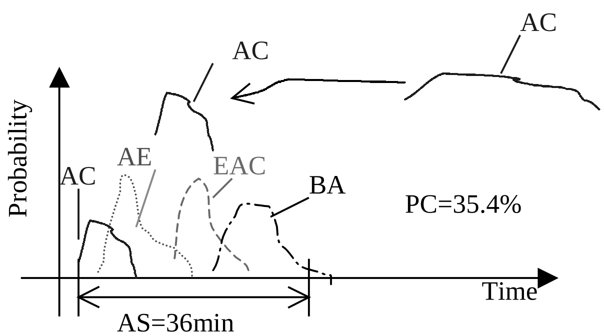


Figure 13: Tactical planning

To estimate the advantages of this real time system, the probability that each of the arrival order occurs is computed. For instance, it is not likely that Air Canada arrives before Aer Lingus. Although both flights are planned to land at 8:00 a.m., the mean delay of Air Canada is 20 minutes while Aer Lingus usually arrives 3 minutes earlier.
If the Air Canada flight arrives at the transmitting point (in this case the point **P** placed 1000 Km away from Glasgow International airport) before $t_{PB}^{d}(\zeta) < 6:34$ a.m., there is a high probability that the aircraft will arrive early and thus it will be accommodate as the first in the landing order. If the Air Canada flight arrives at the transmitting point between 6:34 a.m. and 6:45 a.m., it will be the second in the arrival order having a high probability of arriving after Aer Lingus. European Air Charter will then be shifted into the third position and ground hold delay will be allocated to it.

If the Air Canada flight arrives before Aer, no ground hold has to be imposed upon the other flights as they are already planned to arrive simultaneously. As shown in Table 5, the probability of this happening is 1.9%. If the flight from Toronto arrives at the point **P** located 1000 Km away from the airport between 6:34 a.m. and 6:47 a.m., there is a probability of 25.2% that the Air Canada and Aer Lingus flights will arrive at the same time (see Figure 11 and Table 5). To avoid collision, the other last three flights require to be shifted as many minutes as Air Canada arrives at that point after 6:34 a.m. (see Table 6). For instance, if it crosses that point at 6:40 a.m., a ground hold delay of 6 minutes has to be imposed to the other flights



Reduction by 35.7%.
This event occurs with probability equals to 25.2%

Figure 11: Tactical planning: Air Canada arrives before Aer Lingus

Arrival order	Probability of the Event	Time Condition
AC…AE,EAC,BA	1.9%	t^{d}_{PB} <6:34
AC,AE,EAC,BA	25.2%	6:34≤ t^{d}_{PB} <6:47
AE,**AC**,EAC,BA	24.9%	6:47≤ t^{d}_{PB} <6:55
AE,EAC,**AC**,BA	18.0%	6:55≤ t^{d}_{PB} <7:05
AE,EAC,BA…**AC**	30.0%	t^{d}_{PB} ≥7:05

Table 5: Probability and time condition

There is a probability of 24.9% corresponding to the event: “Air Canada arrives between Aer Lingus and European Air Charter” (see Table 5). In this situation, ground hold delay would be imposed only on the last two flights. The amount of delay is equal to the difference between the arrival time of Air Canada to the reference point and 6:47 a.m (see Table 6).

In 18.0% of cases, Air Canada will be arrive between European Air Charter and British Airways. In this circumstance ground hold delay will be imposed only to the last flight. In the last rows of Table 6 no ground hold is required as Air Canada arrives in the last position after British Airways.

Remember that only the flights until 8:25 a.m. are analysed excluding all those which are planned to come later. To estimate the improvements achieved in term of reduction of PC with this methodology all the flights with which the Air Canada arrival could conflict have to be included. In term of the Arrival Slot (AS) a reduction mean by 32.1 % is achieved, note that in this circumstance the ground hold cost has to be taken into account.

Arrival order	Ground Hold for AE	Ground Hold for EAC	Ground Hold for BA
AC…AE,EAC,BA	0	0	0
AC,AE,EAC,BA	t^{d}_{PB} -6:34	t^{d}_{PB} -6:34	t^{d}_{PB} -6:34
AE,**AC**,EAC,BA	0	t^{d}_{PB} -6:47	t^{d}_{PB} -6:47
AE,EAC,**AC**,BA	0	0	t^{d}_{PB} -6:55
AE,EAC,BA…**AC**	0	0	0

Table 6: Amount of ground hold delay to assign

4. CONCLUSIONS AND FURTHER DEVELOPMENTS

This paper has presented a tool for the development of an operational procedure for strategic and tactical planning. It is based on Statistics and Probability theory and relies on historical data modelling the arrival times as real random variables.

This new procedure has been applied to improve the strategic schedule of Glasgow International Airport using CAA real data. It has been proved that the consequential benefits include the options of:

- Reducing the Arrival Slot length (AS) by 14.3% thereby increasing the airport capacity.
- Reducing the Probability of Conflict (PC) by 15.3% which will decrease airborne delay at the arrival airport, which will reduce fuel consumption both reducing operators costs and environmental pollution.
- A combination of the two options.

Furthermore the tactical planning procedure has been presented and tested always for Glasgow International Airport. The results of the tests have indicated a promising mean reduction of the Arrival Slot (AS) equals to 32.1%. The advantage of such a high percentage is a significant improvement in the airport's capacity whereas the disadvantage is an increase in expenses due to the allocation of ground hold delays.

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6. BIOGRAPHICAL NOTE

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