

# Aircraft relative guidance : a flatness synthesis of a new autopilot mode

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## Abstract

*During the last few years, several concepts concerning the delegation to the flight crew of some tasks currently performed by the air traffic controllers have emerged [3]. Among these new ideas, relative guidance has appeared to be of some interest to contribute to the enhancement of air traffic capacity [1] though it rises hard technical challenges. Indeed, this kind of manoeuvre appears difficult to perform manually, and may induce an excessive increase of the flight crew workload, thus requiring a new on-board automated function, as suggested in [6]. This paper aims at providing some technical insights into an airborne relative guidance control system designed to perform merging manoeuvres and to maintain station keeping behind a designated aircraft. The investigated approach is based on flatness control. This is a recent nonlinear control design approach which is useful in situations where explicit trajectory generation is required [5]. The interest of such an approach is that it tackles the separation between the leading and the trailing aircraft to safely manage the whole manoeuvre. Performances based on a case study including wind are also discussed in this paper. Note that our goal is to show the feasibility, from the pilot's point of view, of an automated relative guidance manoeuvre generator and, though promising, further refinements and extensive validation are indeed needed.*

## 1 Introduction

The anticipated traffic increase and future changing needs in air traffic encourages the design of new strategies to increase air traffic control capacity significantly while at the same time enhancing safety and flight efficiency.

So as to meet this challenge, new concepts such as the delegation to the flight crew of some tasks

presently performed by air traffic controllers have emerged during the last few years [3].

In particular, a subset of this delegation concept concerns the relative guidance of aircraft. The main challenge for aircraft relative guidance is to enhance air traffic capacity by decreasing air traffic controller workload while at the same time preventing flight crew workload increase. To achieve these goals, new automated functions onboard aircraft must be developed; indeed, no automatic control mode is available on-board commercial aircraft to perform this task nowadays.

The relative guidance concept is supported by the European air traffic control agency [3]. Moreover, the station keeping procedure, which is strongly related to relative guidance, is currently investigated in some R&D European projects [1].

Recent studies have investigated related problems for Unmanned Air Vehicles (UAV) [7] and military aircraft [9]. Nevertheless, research for commercial aircraft in this area is just starting: some papers deal with station keeping control: in [1], station keeping is performed manually by the flight deck, whereas in [10] the authors consider a proportional, integral, and derivative (PID) controller to maintain station keeping. But very few papers deal with the automatic control of the merging manoeuvre: in [8], two approaches based on nonlinear control are presented: the first one is based on feedback linearizing control, whereas the second one is based on optimal control, but for both of them the separation between the two aircraft is not taken explicitly into account during the design process. This may result in a loss of separation between the two aircraft during the merging manoeuvre.

In this context, this paper focuses on the design of a nonlinear control system which enables an aircraft to perform merging manoeuvres and to maintain station keeping behind a designated aircraft. The investigated approach is based on flatness control. This is a recent nonlinear control technique which

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is useful in situations where explicit trajectory generation is required [5]. Thus, it allows to control the separation between the two aircraft during the design process.

The relative guidance dynamics are restricted to the horizontal plane, assuming that altitude and flight path angle are controlled by separate autopilots which are decoupled from the velocity and heading dynamics. So, the flight crew is free to move in the vertical plane in case of unexpected situations.

The paper is organized as follows: in the next section, current air traffic control practices in terminal area and anticipated practices are reviewed. Then, the relative motion kinematics are introduced, including reference frame, aircraft model and nonlinear state space representation. The subsequent section presents the design of the flatness-based controller. Then a scenario is proposed to illustrate the performances of the controller, and results from computer simulations are presented. Finally, conclusions are raised.

## 2 Current air traffic control practices in terminal area and anticipated practices

### 2.1 Current practice

For terminal control area (TMA) under radar coverage, the air traffic controller in charge of the sector has two possibilities to guide the aircraft on the runway axis, which is supposed to be equipped with Instrument Landing System (ILS):

- The first one consists in clearing the aircraft on standard approach trajectories, that is to say Standard Arrival (STAR) until Initial Approach Fix (IAF) point, and published approach procedure from IAF to Final Approach Fix (FAF) point. The published approach procedure may include intermediate points, denoted IF, and takes into account several constraints, like departure/arrival strategic separation, military zones, etc.
- The second one consists in guiding the aircraft out of standard approach trajectories, thanks to radar vectoring; the air traffic controller shall then present the aircraft on the localizer beam at a published steady level, and well before glide interception.

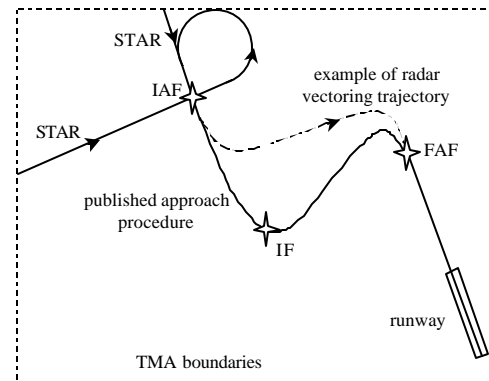


Figure 2-1 Typical route pattern for arriving aircraft

Flying over IAF is generally requested for safety reasons linked with radio failure and terrain collision avoidance.

Generally speaking, air traffic controller makes use of radar vectoring after IAF when a shorter trajectory compatible with altitude to be lost by the aircraft exists, or when inserting the aircraft on a standard published approach procedure is not feasible. For example, this last possibility may happen to enable the runway rate of landing.

### 2.2 Anticipated practice

The anticipated practice covers the so-called “station keeping” concept, which has been investigated since the beginning of the 80s [11].

From the air traffic controller perspective, the operational procedure will basically consist in checking that aircraft is suitably equipped and then requesting the flight crew of the trailing aircraft to engage a manoeuvre in order to merge and/or maintain a given distance or delay with respect to a given aircraft.

From the trailing aircraft flight crew perspective, the assumed procedure when receiving the clearance for relative guidance will firstly consist in positive identification of the leading aircraft (designated by ATC) on a dedicated device, then in checking the feasibility of the manoeuvre and finally in selecting a new autopilot mode which will enable the aircraft to achieve the requested distance or delay.

The feasibility of such a device is based on the ability of each aircraft to broadcast and receive suitable parameters thanks to ADS-B. It includes callsign, position, altitude, ground speed, track and vertical speed [1].

In case of unexpected situation, the flight crew reports to the controller which is assumed to re-establish, if required, the sequencing and handle-back the aircraft.

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Of course, this procedure requires a common agreement between the agents. It implies that the flight crew may refuse the air traffic controller request and that the air traffic controller is not compelled to use the possibilities of the suitably equipped aircraft.

An alternative procedure where the flight crew of the trailing aircraft requests for station keeping may also be considered.

### 3 Relative motion kinematics

#### 3.1 Reference frame

The purpose of the relative guidance control system is first to guide the trailing aircraft towards the leading aircraft and then to maintain the desired position of the former. In order to design this system, the equations of relative motion must be established within an appropriate reference frame.

The here considered reference frame is affixed to the trailing aircraft, as shown in the following figure.

The along track distance, noted TK, is aligned with the trailing aircraft ground speed vector, whereas the cross track distance, noted XTK, is the right handed positive normal distance from the trailing to the leading aircraft. The heading angle of the trailing aircraft is denoted by  $\psi$ , its airspeed by  $V$ . Subscript L is added for all variables related to the leading aircraft. Since wind is considered in this paper, the track angle of the trailing aircraft is denoted by  $\chi$ .

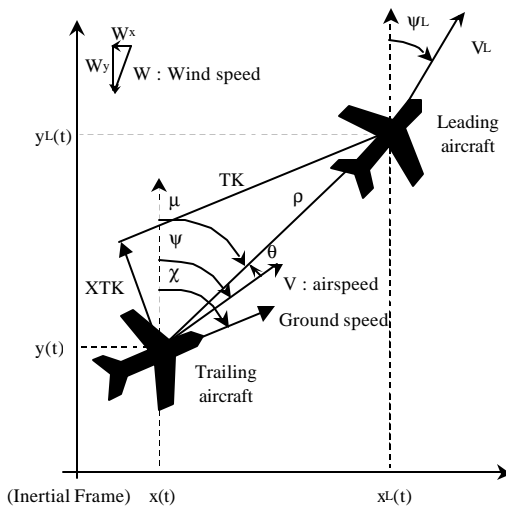


Figure 3-1 Reference frame

Assuming that the earth is flat and non-rotating, it may be considered as an inertial frame. The track angle  $\chi$  is the direction followed by the aircraft with respect to this inertial frame, whereas the heading

angle  $\psi$  is the direction followed by the aircraft with respect to the air. When there is no wind (i.e.  $W_x=W_y=0$ ), those angles are equal.

#### 3.2 Inertial position dynamics

From Figure 3-1, the inertial position dynamics of the trailing aircraft are given by:

$$\begin{cases} \dot{x} = V \cdot \sin(\mathbf{y}) + W_x \\ \dot{y} = V \cdot \cos(\mathbf{y}) + W_y \end{cases} \quad (3-1)$$

Those relations hold even if the motion of the aircraft in the vertical plane is considered as far as the flight path angle  $\gamma$  is small, which is a realistic assumption for commercial aircraft.

Denoting by  $\psi_w$  the wind direction and  $W$  its velocity, and taking into account that the wind direction is the direction *from* where the wind is blowing (so  $\psi_w$  is zero if the wind is blowing *from* the North), the following relation holds:

$$\begin{cases} W_x = W \cdot \sin(\mathbf{y}_w - \mathbf{p}) \\ W_y = W \cdot \cos(\mathbf{y}_w - \mathbf{p}) \end{cases} \quad (3-2)$$

#### 3.3 Relative position dynamics

From Figure 3-1, the relative position of the leading aircraft from the reference frame affixed to the trailing aircraft can be expressed in terms of the inertial positions of the trailing aircraft and leading aircraft:

$$\begin{bmatrix} TK(t) \\ XTK(t) \end{bmatrix} = R(\mathbf{c}) \cdot \begin{bmatrix} x_L(t) - x(t) \\ y_L(t) - y(t) \end{bmatrix} \quad (3-3)$$

where the rotation matrix  $R(\chi)$  is defined by:

$$R(\mathbf{c}) = \begin{bmatrix} \sin(\mathbf{c}) & \cos(\mathbf{c}) \\ \cos(\mathbf{c}) & -\sin(\mathbf{c}) \end{bmatrix}. \quad (3-4)$$

Noticing that  $R^{-1}=R$ , taking into account the inertial position dynamics expressed in ( 3-1 ), and assuming the same wind for the leading and the trailing aircraft, the time derivative of ( 3-3 ) yields:

$$\begin{bmatrix} \dot{TK} \\ \dot{XTK} \end{bmatrix} = \dot{\mathbf{c}} \cdot \begin{bmatrix} XTK \\ -TK \end{bmatrix} + R(\mathbf{c}) \cdot \begin{bmatrix} V_L \cdot \sin(\mathbf{y}_L) - V \cdot \sin(\mathbf{y}) \\ V_L \cdot \cos(\mathbf{y}_L) - V \cdot \cos(\mathbf{y}) \end{bmatrix} \quad (3-5)$$

The track angle  $\chi$  and ground speed  $G_s$  are defined as follows:

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$$\begin{cases} \dot{x} = G_s \cdot \sin(\mathbf{c}) \\ \dot{y} = G_s \cdot \cos(\mathbf{c}) \end{cases} \quad (3-6)$$

Referring to ( 3-1 ) and ( 3-2 ), the track angle  $\chi$  and the heading angle  $\psi$  are linked by the following relationship:

$$\begin{cases} \sin(\mathbf{c}) = \frac{V \cdot \sin(\mathbf{y}) - W \cdot \sin(\mathbf{y}_w)}{G_s} \\ \cos(\mathbf{c}) = \frac{V \cdot \cos(\mathbf{y}) - W \cdot \cos(\mathbf{y}_w)}{G_s} \\ G_s = \sqrt{V^2 + W^2 - 2 \cdot V \cdot W \cdot \cos(\mathbf{y} - \mathbf{y}_w)} \end{cases} \quad (3-7)$$

**3.4 Flight dynamics**

The trailing aircraft is supposed to fly in a fully coordinated fashion, i.e. the side-slip angle is always zero (airspeed and fuselage have the same direction).

Furthermore it is assumed that the following two decoupled autopilot functions are available onboard the trailing aircraft. These decoupled functions assume coordination between throttle, aileron and rudder, as in many modern jets.

- The airspeed hold autopilot controls the conventional airspeed  $V$  without affecting the aircraft's altitude. Denoting by  $V_c$  the controlled airspeed, the airspeed dynamics may be modeled as a first order system for the purpose of control design:

$$\dot{V} = \frac{V_c - V}{t_v} \quad (3-8)$$

- The heading hold autopilot controls the heading  $\psi$  without affecting the aircraft's airspeed. Heading is assumed to be controlled by the bank angle  $\phi$ . For small bank angle and loading factors, the following relation between heading rate and bank angle holds, where  $g$  is the acceleration of gravity,  $\phi$  the bank angle and  $V$  the actual airspeed:

$$\dot{\psi} = \frac{g \cdot \phi}{V} \quad (3-9)$$

Denoting by  $\phi_c$  the controlled bank angle, the bank angle dynamics may be modeled as a first order for the purpose of control design:

$$\dot{\phi} = \frac{\phi_c - \phi}{t_\phi} \quad (3-10)$$

These first-order models of the airspeed and bank angle are usually considered as good models for inner loops flight dynamics [9].

**3.5 State space representation**

Gathering ( 3-4 ), ( 3-5 ), ( 3-8 ) and ( 3-9 ), a state space representation of the relative guidance kinematics is:

$$\begin{cases} \dot{TK} = \dot{\mathbf{c}}(V, \dot{V}, \mathbf{y}, \dot{\mathbf{y}}) \cdot XTK \\ -V \cos(\mathbf{y} - \mathbf{c}) + V_L \cos(\mathbf{y}_L - \mathbf{c}) \\ \dot{XTK} = -\dot{\mathbf{c}}(V, \dot{V}, \mathbf{y}, \dot{\mathbf{y}}) \cdot TK \\ -V \sin(\mathbf{y} - \mathbf{c}) + V_L \sin(\mathbf{y}_L - \mathbf{c}) \\ \dot{\mathbf{y}} = g \cdot \mathbf{j} / V \\ \dot{V} = (V_c - V) / t_v \end{cases} \quad (3-11)$$

where the expression  $\dot{\mathbf{c}}(V, \dot{V}, \mathbf{y}, \dot{\mathbf{y}})$  is obtained by differentiating ( 3-7 ). Note that it also depends on the wind characteristics  $(W, \dot{W}, \mathbf{y}_w, \dot{\mathbf{y}}_w)$  that are generally available on-board through the Air Data Computer (ADC).

Denoting by  $\underline{u}$  the control vector and by  $\underline{x}$  the state vector, the previous representation reduces to:

$$\begin{cases} \dot{\underline{x}} = f(\underline{x}, \underline{u}) \\ \underline{x} = [TK \quad XTK \quad \mathbf{y} \quad V]^T \\ \underline{u} = [V_c \quad \mathbf{j}]^T \end{cases} \quad (3-12)$$

The previous state space representation will be used for the controller design. In this representation, the dynamics ( 3-10 ) of the controlled bank angle  $\phi_c$  are not taken into account since they are much faster than the relative position dynamics (nevertheless, they are taken into account during simulations).

**4 Controller design****4.1 Differentially flat systems**

Flatness was originally introduced in [4]. Roughly speaking, a flat system is a square input/output system (i.e. a system having the same number of inputs and outputs) for which there exists an output vector such that all states and inputs can be expressed in terms of this output vector and its derivatives.

More precisely, a nonlinear system:

$$\dot{\underline{x}} = f(\underline{x}, \underline{u}), \quad \underline{x} \in \mathfrak{R}^n, \underline{u} \in \mathfrak{R}^m \quad (4-1)$$

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is differentially flat if one can find an output  $\underline{z} \in \mathfrak{R}^m$  of the form:

$$\underline{z} = \mathbf{V}(\underline{x}, \underline{u}, \dots, \underline{u}^{(s)}) \quad (4-2)$$

where  $\underline{u}^{(s)}$  denotes the  $s$ -th order derivative of  $\underline{u}$  with respect to time, and such that:

$$\begin{cases} \underline{x} = \mathbf{J}_0(\underline{z}, \dots, \underline{z}^{(r)}) \\ \underline{u} = \mathbf{J}_1(\underline{z}, \dots, \underline{z}^{(r)}, \underline{z}^{(r+1)}) \end{cases} \quad (4-3)$$

Such a  $\underline{z}$  output is called flat output.

In addition, system ( 4-1 ) is said Lie-Bäcklund equivalent to the following system (called trivial system), where  $\underline{v}$  is the new input:

$$\dot{\underline{v}} = \underline{z}^{(r+1)} \quad (4-4)$$

Differentially flat systems are useful in situations where explicit trajectory generation is required. Since the behavior of flat systems is determined by their flat outputs, trajectory in the flat output space can be planned and then mapped to appropriate inputs.

## 4.2 Choice of flat outputs

In the following,  $\rho$  denotes the (horizontal) range between the leading and the trailing aircraft, and  $\mu$  the relative bearing between those aircraft. They are related to the state variables of ( 3-12 ) by:

$$\begin{cases} \mathbf{r} = \sqrt{TK^2 + XTK^2} \\ \mathbf{m} = \mathbf{c} + \arctg\left(\frac{XTK}{TK}\right) \end{cases} \quad (4-5)$$

These variables are of major interest for the achievement of relative guidance. In addition, they are similar to those provided by the TCAS<sup>1</sup> surveillance, or could be derived from future Automatic Dependent Surveillance Broadcast (ADS-B) systems [2].

The purpose of this section is to show that the pair  $(\rho, \mu)$  is a flat output vector of system ( 3-11 ).

First of all, the following relation between the leading aircraft's inertial position and the trailing aircraft's one is immediate from *Figure 3-1*:

$$\begin{cases} x = x_L - \mathbf{r} \cdot \sin(\mathbf{m}) \\ y = y_L - \mathbf{r} \cdot \cos(\mathbf{m}) \end{cases} \quad (4-6)$$

So the goal is, according to ( 4-6 ), to find the relationship relating the state vector  $\mathbf{x}$  and the control vector  $\underline{u}$  defined in ( 3-12 ) to the flat output components  $\rho$  and  $\mu$  and their derivatives. For that purpose, it is assumed that the leading aircraft and wind characteristics, i.e.  $(x_L, y_L, V_L, \mathbf{y}_L, W, \mathbf{y}_w)$ , are available on-line. Moreover, we assume that the trailing and the leading aircraft are subject to the same wind.

Taking into account ( 3-1 ) and differentiating ( 4-6 ) with respect to time leads to:

$$\begin{cases} V \cdot \sin(\mathbf{y}) = V_L \cdot \sin(\mathbf{y}_L) \\ \quad - \dot{\mathbf{r}} \cdot \sin(\mathbf{m}) - \mathbf{r} \cdot \dot{\mathbf{m}} \cdot \cos(\mathbf{m}) \\ V \cdot \cos(\mathbf{y}) = V_L \cdot \cos(\mathbf{y}_L) \\ \quad - \dot{\mathbf{r}} \cdot \cos(\mathbf{m}) + \mathbf{r} \cdot \dot{\mathbf{m}} \cdot \sin(\mathbf{m}) \end{cases} \quad (4-7)$$

This leads to the expression of airspeed and heading:

$$\begin{aligned} V^2 &= V_L^2 + \dot{\mathbf{r}}^2 + \mathbf{r}^2 \dot{\mathbf{m}}^2 \\ &\quad - 2V_L \dot{\mathbf{r}} \cos(\mathbf{y}_L - \mathbf{m}) \\ &\quad - 2V_L \mathbf{r} \dot{\mathbf{m}} \sin(\mathbf{y}_L - \mathbf{m}) \end{aligned} \quad (4-8)$$

and:

$$\tan(\mathbf{y}) = \frac{V_L \sin(\mathbf{y}_L) - \dot{\mathbf{r}} \sin(\mathbf{m}) - \mathbf{r} \dot{\mathbf{m}} \cos(\mathbf{m})}{V_L \cos(\mathbf{y}_L) - \dot{\mathbf{r}} \cos(\mathbf{m}) + \mathbf{r} \dot{\mathbf{m}} \sin(\mathbf{m})} \quad (4-9)$$

Thus, airspeed and heading are clearly expressed in terms of  $(\mathbf{r}, \mathbf{m}, \dot{\mathbf{r}}, \dot{\mathbf{m}})$ .

Furthermore, relation ( 3-7 ) leads to:

$$\tan(\mathbf{c}) = \frac{V \sin(\mathbf{y}) - W \sin(\mathbf{y}_w)}{V \cos(\mathbf{y}) - W \cos(\mathbf{y}_w)} \quad (4-10)$$

Combiné with ( 4-8 ) and ( 4-9 ), we conclude that  $\chi$  is also function of  $(\mathbf{r}, \mathbf{m}, \dot{\mathbf{r}}, \dot{\mathbf{m}})$ .

Finally, referring again to *Figure 3-1*, the cross-track and the along-track distances read:

$$\begin{cases} TK = \mathbf{r} \cdot \cos(\mathbf{m} - \mathbf{c}) \\ XTK = \mathbf{r} \cdot \sin(\mathbf{m} - \mathbf{c}) \end{cases} \quad (4-11)$$

Combined with ( 3-8 ) and ( 3-9 ), this shows that all the system variables can be expressed as a function of  $(\rho, \mu)$  and their first and second order derivatives: it achieves to prove that  $(\rho, \mu)$  constitute a flat output vector for system ( 3-11 ).

<sup>1</sup>Traffic Collision Avoidance System

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### 4.3 Merging trajectory planning

It is assumed that at the starting time  $t_0$  of the relative guidance maneuver, the data  $\rho_0$  and  $\mu_0$  are available, and that at final time the desired values  $(\mathbf{r}_c, \mathbf{m}_c)$  are specified.

Thus, a reference trajectory  $t \rightarrow (\mathbf{r}_{ref}(t), \mathbf{m}_{ref}(t))$  for range and bearing can be obtained as follows, where  $\tau_p$  and  $\tau_\mu$  are adequate time constants:

$$\begin{cases} \mathbf{r}_{ref}(t) = \mathbf{r}_c + (\mathbf{r}_0 - \mathbf{r}_c) \exp(-t/\tau_r) \\ \mathbf{m}_{ref}(t) = \mathbf{m}_c + (\mathbf{m}_0 - \mathbf{m}_c) \exp(-t/\tau_m) \end{cases} \quad (4-12)$$

The main motivation for choosing such exponential behavior for the flat outputs is that their derivatives do not depend explicitly on time. So, the computations can be made on line without the necessity to store the starting time of the maneuver. Indeed, the  $n^{\text{th}}$  derivative of the reference range and relative bearing can be expressed as:

$$\begin{cases} \mathbf{r}_{ref}^{(n)}(t) = \left( \frac{-1}{\tau_r} \right)^n \cdot (\mathbf{r}_{ref}(t) - \mathbf{r}_c) \\ \mathbf{m}_{ref}^{(n)}(t) = \left( \frac{-1}{\tau_m} \right)^n \cdot (\mathbf{m}_{ref}(t) - \mathbf{m}_c) \end{cases} \quad (4-13)$$

It is worth noting that the reference trajectory may be freely chosen without taking into account the perturbations that might affect the system, as opposed to the feedback. Therefore, it is chosen differentiable, first, for simplicity's sake and second, because, as a consequence of flatness, the trajectories that the system can readily follow naturally belong to this class.

### 4.4 Closed loop tracking

A consequence of the flatness of system (3-11) is that it is Lie-Bäcklund equivalent to the following system:

$$\begin{cases} \ddot{\mathbf{r}} = \mathbf{v}_1 \\ \ddot{\mathbf{m}} = \mathbf{v}_2 \end{cases} \quad (4-14)$$

Since (4-14) indicates that the second derivatives of  $\rho$  and  $\mu$  can be interpreted as new controls, they are chosen as follows:

$$\begin{cases} \mathbf{v}_1 = \ddot{\mathbf{r}}_{ref} - 2\mathbf{x}_r w_r (\dot{\mathbf{r}} - \dot{\mathbf{r}}_{ref}) - w_r^2 (\mathbf{r} - \mathbf{r}_{ref}) \\ \mathbf{v}_2 = \ddot{\mathbf{m}}_{ref} - 2\mathbf{x}_m w_m (\dot{\mathbf{m}} - \dot{\mathbf{m}}_{ref}) - w_m^2 (\mathbf{m} - \mathbf{m}_{ref}) \end{cases} \quad (4-15)$$

The damping ratios  $\xi_p$  and  $\xi_\mu$  have been set to 1 to enforce smooth second order dynamics for the flat outputs. Indeed, inserting (4-14) into (4-15) and solving the resulting second order differential equations for the range leads to:

$$\mathbf{r}(t) = \mathbf{r}_{ref}(t) + (\dot{\mathbf{r}}_0 - \dot{\mathbf{r}}_{ref}(0)) \cdot t \cdot \exp(-w_r t) \quad (4-16)$$

The difference  $(\mathbf{r}(t) - \mathbf{r}_{ref}(t))$  has the following shape:

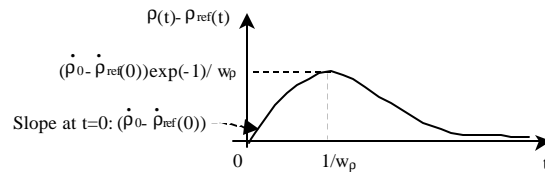


Figure 4-1 Difference between the actual and the desired range

This will remain positive as far as the following condition is satisfied:

$$\mathbf{r}_c < \mathbf{r}_0 + \tau_r \dot{\mathbf{r}}_0 \quad (4-17)$$

As a consequence, the separation between the leading and the trailing aircraft will be safely managed during all the manoeuvre provided that the range and its derivative satisfy condition (4-17) at the beginning of the manoeuvre.

Given the positions and velocities of the trailing and leading aircraft, the outputs are computed as follows: the values of  $(\mathbf{r}, \mathbf{m}, \dot{\mathbf{r}}, \dot{\mathbf{m}})$  are firstly computed thanks to (4-5) and its derivative. Then, these values are used with (4-12) and (4-13) into (4-15) to set the values of the desired flat outputs. Finally, the controlled bank angle and airspeed are derived.

## 5 Case study

### 5.1 Scenario

In this section, a scenario is designed in order to evaluate the properties of the control laws previously designed.

In this scenario, the leading aircraft acceleration and heading rate are not broadcasted; consequently the leading aircraft speed and heading are taken as

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constants in the relative guidance controller (i.e.  $\dot{V}_L = 0 ; \dot{Y}_L = 0$ ). So, a special attention is given on the behaviour of such a controller to the changes of heading and airspeed from the leading aircraft.

The leading aircraft starts at  $x_0 = 0$  NM,  $y_0 = 0$  NM, with initial conventional airspeed and heading of 240 kts and 90 degrees respectively. It is supposed to broadcast its data every second.

The controlled bank angle of the leading aircraft is always zero, except between 600 sec and 630 sec where the leader changes its heading of about 50 degrees with a bank angle of 20 degrees.

The controlled conventional airspeed of the leading aircraft is first set at 240 kts for  $t \leq 300$  sec, and then is set to 190 kts.

The trailing aircraft starts at  $x_0 = +10$  NM,  $y_0 = -10$  NM, with initial conventional airspeed and heading of 240 kts and 0 degrees respectively.

The simulation period lasts 15 min (900 sec), and the requested separation for the trailing aircraft remains constant and equal to 5 NM behind the leading aircraft.

The whole manoeuvre is supposed to take place within a wind of 20 kts blowing from North.

During the manoeuvre, the inputs (i.e. the controlled bank angle and airspeed) of the relative guidance controller are limited to the following 'safe' values:

$$\begin{cases} -20 \text{ deg.} \leq j_c \leq +20 \text{ deg.} \\ 170 \text{ kts} \leq V_c \leq 250 \text{ kts} \end{cases} \quad (5-1)$$

Furthermore, roll rate and acceleration are limited to the following values:

$$\begin{cases} |j| \leq +5 \text{ deg./sec} \\ |\dot{V}| \leq 1 \text{ kts/sec} \end{cases} \quad (5-2)$$

Those limitations modify condition ( 4-17 ) but the induced modifications are out of the scope of this paper.

The time constants  $\tau_v$  and  $\tau_\phi$  of the airspeed and heading hold autopilot are set to the following values:

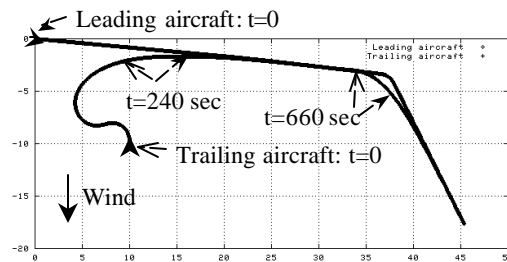
$$\begin{cases} t_v = 40 \text{ sec} \\ t_j = 5 \text{ sec} \end{cases} \quad (5-3)$$

The values of the constants defining the reference trajectories and the tracking performances have been set as follows:

$$\begin{cases} t_r = t_m = 50 \text{ sec} \\ w_r = w_m = 0.03 \text{ rad/sec} \\ x_r = x_m = 1 \end{cases} \quad (5-4)$$

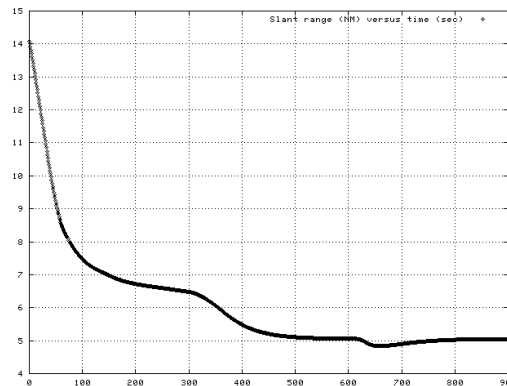
**5.2 Results**

The movements of the leading and trailing aircraft in the horizontal (orthonormed coordinates) plane are shown in *Figure 5-1*. The curvature of the trajectory at the beginning of the manoeuvre comes from the saturation of the controlled variables. Due to the proximity of the two aircraft, a negative controlled airspeed would be required to move the trailing aircraft away from the leader; nevertheless, according to the limitations, the trailing aircraft moves at minimum airspeed and maximum negative bank angle (see *Figure 5-4* and *Figure 5-5*). This feature clearly shows the tendency of the controller to safely manage the separation between the two aircraft.



*Figure 5-1 Aircraft movement in the horizontal plane (axes in NM)*

The evolution in the actual range between the leading and the trailing aircraft is shown in *Figure 5-2*. As intended, the slant range tends to the desired 5 NM. Furthermore, the feedback tends to stick to that desired value despite the changes in leading aircraft heading and airspeed. This shows the robustness of this approach.



*Figure 5-2 Actual range (in NM) between the leading and the trailing aircraft versus time (in sec)*

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The next figure shows the exponential behaviour of the relative bearing, as expected from expression (4-12).

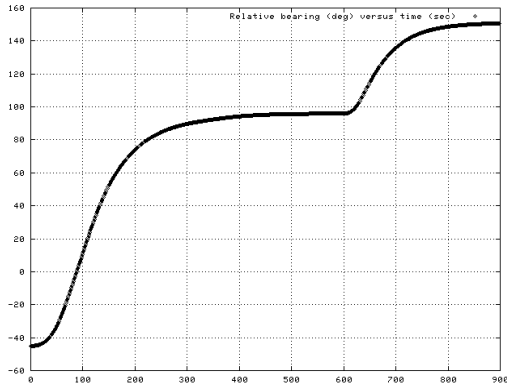


Figure 5-3 Relative bearing (in deg) between the leading and the trailing aircraft versus time (in sec)

The time response of the relative bearing, about 300 sec, is quite long: this can be explained by the presence of saturations in the controlled airspeed and bank angle, as shown in Figure 5-4 and Figure 5-5:

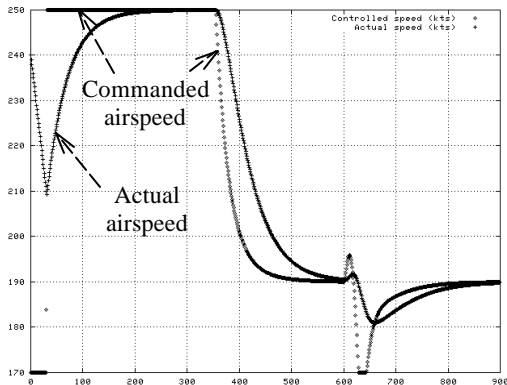


Figure 5-4 Controlled airspeed (in kts) versus time (in sec)

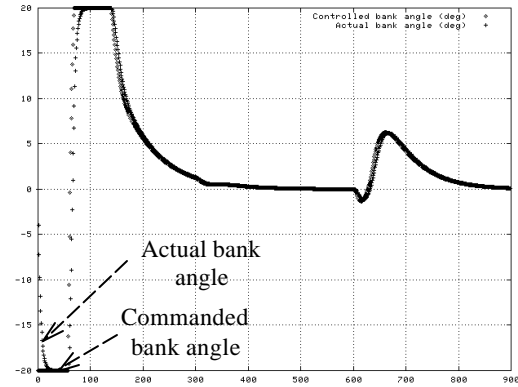


Figure 5-5 Controlled and actual bank angle (in degrees) versus time (in sec)

As the previous one, this last figure shows the usefulness to set a reference trajectory for the range and relative bearing in order to avoid abrupt changes in the controls.

Furthermore, it shows that neglecting the bank angle dynamics during the design of the controller does not have a significant impact: indeed, it just delays the actual bank angle.

Finally, the following figure shows the load factor as a function of time. It shows that the maneuver remains quite comfortable for the passengers:

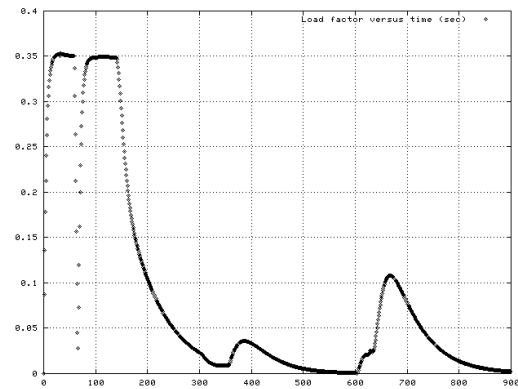


Figure 5-6 Load factor versus time (in sec)

**6 Conclusion**

In this paper, the design of a combined feedforward and feedback law to achieve relative guidance manoeuvres has been considered.

The proposed approach is based on flatness. One of the key-point of such a design is that the trailing aircraft is driven along a reference trajectory computed on line. The proposed design of the reference trajectory relies on exponential functions, but other approaches are possible, such as polynomial interpolation. In addition, the reference trajectory computed on line takes into account the



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separation between the leading and the trailing aircraft in order to safely manage the whole manoeuvre.

As far as available on-line information and communications are concerned, better performances may be achieved if the leading aircraft acceleration and heading rate are available to the follower: such data may be broadcasted, or computed onboard the trailing aircraft thanks to an observer.

The paper has focused on a separation objective expressed in terms of distance. Nevertheless, some simulations have shown the interest to express the separation objective in terms of delay [1]. So, provided that the delay criteria is translated into a separation objective, the proposed design can be easily extended to this case.

This approach appears quite promising. It deserves further studies, especially on the robustness of such a controller to noisy data from the leading aircraft and to wind gusts. In addition, special attention may be paid to neglected dynamics.

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## 8 Biographies

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## 9 Key words

TMA: terminal control area

ASAS: Airborne Separation Assistance System