

# A Market Mechanism to Assign Air Traffic Flow Management Slots

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**Abstract**—We propose a market mechanism based on auctions, which could constitute an efficient tool to assign Air Traffic Flow Management delays to flights at a tactical level, when a mismatch between demand and capacity is detected for a specific system resource. Such a mechanism constitutes an improvement to the monolithic central allocation employed today, because the current First Planned First Served solution is taken as the baseline from which a more efficient one is iteratively searched in the solution space. We prove that each actor is better-off when the mechanism converges to an optimal solution. This mechanism actively involves users in the decision making process while respecting at the same time the concepts of transparency, equity, efficiency and non disclosure of airlines private information, as advocated by the User Driven Prioritization Process in the SESAR Target Concept.

**Keywords**—Auctions; Demand Capacity Balance; Air Traffic Flow Management slots; Collaborative Decision Making.

## I. INTRODUCTION

The Air Transportation System (ATS) both in Europe and in the U.S. is highly capacity constrained due to the limited availability of resources on the ground and en-route. The capacity of an airport is dependant on the combined availability of its limiting components, such as runways, aircraft parking positions, gates, passenger terminal throughput. A good management of these areas determines the extent to which the airport can reach its full capacity potential. En-route sectors of airspace also have a limited capacity determined by the maximum workload acceptable for the Air Traffic Controllers (ATCOs) [1]. When occasional events occur, either unexpected such as meteorological phenomena and technical failures or predicted in advance such as ATCOs strikes, resource capacity is further reduced.

In the context of Air Traffic Management (ATM) when an imbalance between forecasted traffic and available capacity is detected, it is usually the ATM authority that imposes a regulation, which aims at protecting the potentially overloaded node by imposing delay on some flights. The flights are usually prioritized on a First Planned First Served (FPFS) basis, meaning that the flight which planned to use the resource earlier receives priority on another flight which planned to use it later. In this way delay is imposed without regarding users' preferences, but just on the base of a generally accepted concept of equity among users. The per-minute cost of delay

experienced by a flight can vary within a wide range of values depending on several factors. We demonstrate that in the case of a single capacitated resource, the FPFS criterion produces an optimal allocation in the case of identical cost of delay values. As soon as we introduce different cost weights for the delayed flights, the FPFS solution is no longer optimal and another system must be employed to guarantee an efficient allocation that minimizes the aggregated cost of delay. We propose a decentralized market based mechanism which actively involves Aircraft Operators in the delay allocation process, through the use of auctions. Airlines decide for each flight if it is preferable to acquire resources at the current market price or to accept the delay.

Our research is motivated by the Single European Sky ATM Research program (SESAR), which foresees in the Target Concept that airspace users will be fully involved in the process of demand and capacity balancing. This will be achieved through the implementation of ad-hoc Collaborative Decision Making (CDM) processes, both at a strategic phase, by agreements on how traffic demand or individual trajectories will be adjusted if ANSP and Airports cannot provide sufficient capacity, and at a tactical one through the User Driven Prioritisation Process (UDPP) designed to prioritize traffic queues caused by unexpected capacity shortfalls [2]. In particular, according to the SESAR Target Concept, airspace users among themselves can recommend to the Network Management a priority order for flights affected by delays caused by an unexpected reduction of capacity. This requirement, coupled with the observation that a considerable amount of unnecessary delay is produced each year because of the lack of incentives for airlines to efficiently use assigned ATFM slots, motivates our research. In the following we formulate an auction mechanism that could be employed to optimally assign slots to flights. Section II surveys the most relevant literature on slot allocation policies. Section III provides an overview of the system currently used in Europe to assign ATFM delays. In Section IV we introduce the models that allow to minimize respectively the aggregated delay and the aggregated cost of delay from a central ATM authority perspective, in Section V we develop the mathematical formulation of the market mechanism from a centralized model to a distributed one, followed in Section VI by two computational examples based on real operational

data. Section VII discusses the implications of the results and the future work.

## II. RELATED RESEARCH

Optimization models for the optimal allocation of Air Traffic Flow Management (ATFM) delays were first conceptualized by [3], followed by [4] which formally introduced a stochastic programming model for a single-airport ground holding problem and by [5] which extended the problem to the multi-airport case. A further extension to en-route constraints and re-routing options is due to the ILP model in [6], subsequently evolved in a more compact formulation in [7]. A number of other models dealing with the optimization of ATFM delay allocation appeared in the 1990s, the interested reader can refer to [8]. The common characteristic of all these models is the single decision making authority (the provider of ATFM services) attempting to optimize a “global” objective function obtained by aggregating the direct operating cost caused to individual flights by ATFM measures. However in a real operational environment the Airlines are in the best position to take decisions for their own flights, because they have the necessary information and for this reason they should be involved in the decision loop. This principle has induced both U.S. and European ATM regulators to introduce in the last years a CDM program, whose fundamental idea is to move away from a monolithic central decision maker to “real-time” decision support tools that assist air traffic managers and Airline Operational Centers in making the best decisions based on the most updated information they have. The different nature of the ATFM problem in the U.S. and in Europe has brought to different CDM applications. In the U.S. in fact ATFM is primarily driven by airport capacity constraints, thus allowing Airlines, after a first allocation by the ATFM authority, to decide how to use their allocated share of capacity at each Airport. In Europe on the contrary both airports and en-route ATC sectors of airspace constitute a major bottleneck, thus practical applications of the CDM concept have been mainly concentrated so far at individual Airports, as an efficient way to monitor and optimize turn around processes [9].

Auctions can provide a tool to efficiently assign limited resources to the bidder who values them the most, thus producing an allocation that maximizes the system benefit while taking into account users’ preferences, without requiring them to disclose private information. The use of auctions for assigning airport slots was first proposed by [10] under the form of a sealed-bid combinatorial auction, which permits airlines to submit various contingency bids for flight-compatible combinations of individual airport landing or take-off slots, priced by a central authority in order to maximize the system surplus, as revealed by the set of package bids submitted. After an initial allocation is achieved in a primary market, a secondary market provides the opportunity to buy or sell units that were allocated in primary market. One of the major draw-

backs of their mechanism is that it is not generally incentive compatible, i.e. any bidder may strategically manipulate some bids to get resources at a lower price than the price at which it actually values them. Reference [11] suggests the use of combinatorial auctions for the assignment of airport slots in the U.S., since the items auctioned are scarce commodities with both a private and a common values there is the need to adopt a system of price discovery as the one represented by auctions. They provide slot auction basic principles of design for the U.S system. Reference [12] formalizes a capacity resource market for air traffic flows, in which different competitive airlines have private preference information over traffic control actions. Efficient resource allocation in this market is guaranteed under the assumption that airlines behave as price-taking agents, i.e. airlines take prices as fixed without attempting to game the system, for example by estimating the effect of their bids on market prices.

## III. THE CURRENT EUROPEAN RESOURCE ALLOCATION SYSTEM

In the context of European ATM when an imbalance between demand and availability of air transport infrastructure is detected, the EUROCONTROL Central Flow Management Unit (CFMU) in close collaboration with the regional Flow Management Positions (FMPs), may employ a number of measures at the pre-tactical stage to avoid congestion. These measures can vary from different Air Traffic Control (ATC) sector configurations to enhance capacity, to the activation of mandatory routes for certain trajectories, to the creation of slot allocation regulations. In this latter case an ATFM regulation (also simply referred as regulation) aims at protecting a certain element of the system by limiting the maximum number of flights which can enter it, during an established period of time; thus on the day of operations all flights affected by regulations can either decide to re-route, in order to avoid the affected areas, or to be issued a Controlled Take Off Time (CTOT), represented by an ATFM departure slot. This measure is based on the principle that delays on the ground are safer and less costly than those in the air, thus it is preferred to anticipate any delay forecasted somewhere in the system, at the departure airport prior to the take-off [13]. The ATFM slots are calculated by the Computer Assisted Slot Allocation (CASA) system at CFMU on a FPFS basis [14]. To do so CASA creates for each regulated point, area or airport a Slot Allocation List (SAL) composed by a number of slots depending on the rate of acceptance and the duration of the regulation and assigns consecutively slots to flights as close to their Estimated Time Over (ETO) as available. If two flights require the same slot, it is assigned to the one with the lower ETO. If a flight is affected by several regulations, the delay caused by the most penalizing one is forced in all the others. Delays caused by ATFM measures in 2007 amounted to 21.5M minutes, causing an estimated cost of €1300M to the users [15].

Even though a flight may cross several regulations during its trajectory, most of the ATFM delays are caused by just one, meaning that there is a unique capacity constrained

resource the flight planned to use. Figure 1 displays the number of regulations each regulated flight was affected by, during AIRAC 311 (i.e. the 28 days period from 31<sup>st</sup> July 2008 to 27<sup>th</sup> August 2008).

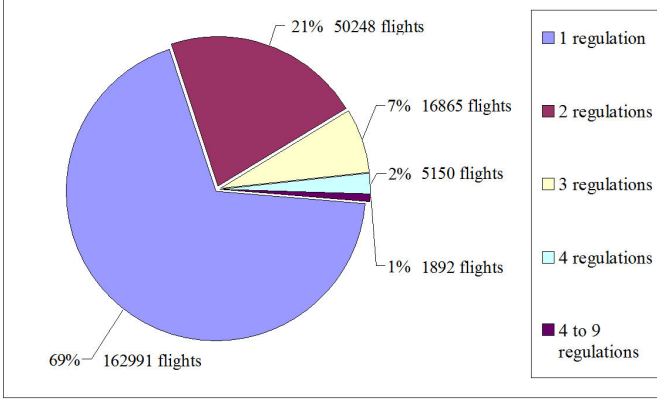


Fig. 1. Flights delayed by number of regulations during AIRAC 311

If we further restrict our scope to the cases in which all flights affected by a specific regulation are affected only by that one, than all these flights compete for the same and unique resource utilization. In those cases the use of multiple goods auctions may constitute an efficient way to allocate slots to flights, with the aim of assigning them to the bidder valuing it most at a price unknown a-priori. In the following we formulate an auction mechanism that could be employed in these specific cases to optimally assign slots to flights.

#### IV. THE CENTRAL SLOT ALLOCATION SYSTEM

Let us consider a set of flights  $F = \{1, \dots, F\}$ , a set of time periods  $T = \{1, \dots, T\}$ , and a single capacity-constrained airport or sector  $s$  they planned to use, with a given fixed capacity associated ( $CAP_s$ ), expressed in number of entries per hour, active from  $st\_time_s$  to  $end\_time_s$ . We can create the “Slot Allocation List” (SAL) for  $s$ , which contains  $RC_s$  slots of capacity 1, where:

$$RC_s = \left\lfloor \frac{end\_time_s - st\_time_s}{\frac{60}{CAP_s}} \right\rfloor$$

Each slot in the SAL begins at a time which is calculated as follows:

$$t_j^B = \left\lfloor st\_time_s + (j - 1) \cdot \frac{60}{CAP_s} \right\rfloor \quad \text{with } j \in \{1, \dots, RC_s\}$$

hence each slot  $I_j$  goes from minute  $t_j^B$  to  $t_j^E \equiv (t_{j+1}^B) - 1$ . It follows that  $I_j \cap I_k = \emptyset$  if  $j \neq k$ , and  $\bigcup I_j = (end\_time_s - st\_time_s)$ .

Each flight  $f \in F$  has a published trajectory which contains the estimated time of entry into  $s$ , indicated as  $ETO_f$ . The FPFS mechanism greedily takes flights in an order determined by an ascending value of  $ETO_f$  and assigns the first available slot  $I_j$  with  $j \in \{1, \dots, RC_s\}$ , subject to the constraint  $ETO_f \leq t_j^E$ . Each slot cannot be assigned to more than 1

flight. Note that we have restricted our scope to the case of a unique capacitated resource  $s$ , so we assume no interactions among different regulations.

An objective of the ATM System Regulator (SR) is to maximize the social welfare by minimizing the total delay in the system [15]. If only one congested sector  $s$  exists, the target is achieved through the following Model (1):

$$\min \sum_{f \in F} \left( \sum_{t \in T} tx_{ft} - ETO_f \right) \quad (1a)$$

$$\sum_{f \in F} \sum_{t \in I_j} x_{ft} \leq 1 \quad \forall j \quad (1b)$$

$$\sum_{t \in T} tx_{ft} \geq ETO_f \quad \forall f \quad (1c)$$

$$\sum_{t \in T} x_{ft} = 1 \quad \forall f \quad (1d)$$

$$x_{ft} \in \{0, 1\} \quad \forall f \in F, t \in T \quad (1e)$$

where the binary variable  $x_{ft}$  is equal to 1 if and only if flight  $f$  enters sector  $s$  at time  $t$ , and 0 otherwise. The objective (1a) is to minimize the cumulative delay subject to the constraints that only one flight can enter  $s$  during each slot of the SAL (1b), each flight cannot enter  $s$  before its  $ETO_f$  (1c), and all flights affected by the regulation have to find an available slot of the SAL (1d). We define here  $x$  as a vector whose components are the variables  $x_{ft} \forall f \in F, \forall t \in T$ . We denote as  $x^C$  the slot assignment obtained by means of the FPFS policy.

*Theorem 1:* A vector  $x^C$  obtained respecting the FPFS rule is an optimal solution for Model (1).

*Proof:* A vector  $x^C$  following the FPFS rule satisfies all constraints (1b) - (1e) and thus is a feasible solution for Model (1). Let  $x^*$  be an optimal solution of Model (1).

Part A) We first prove that if a slot  $j$  is non used in  $x^C$  it must be empty also for  $x^*$ . Let  $t_j^{CB}$  and  $t_j^{CE}$  be the first and last minute of the empty slot  $j$  for solution  $x^C$ , respectively. Each flight  $f$  whose  $ETO_f > t_j^{CE}$  cannot fill slot  $j$  in the optimal solution  $x^*$  due to constraints (1c). Then an empty slot  $j$  in  $x^C$  could be possibly filled only by a flight allocated at time  $t < t_j^{CB}$ . But by filling the empty slot  $j$ , another earlier slot  $k$  (i.e.,  $t_k^{CE} < t_j^{CB}$ ) becomes free. If slot  $k$  cannot be filled by any other flight the new solution is non-optimal because a higher delay is introduced with respect to  $x^C$ . If slot  $k$  is filled by anticipating a flight from another slot  $h$  such that  $t_k^{CB} < t_h^{CB} < t_j^{CB}$  the decrease of the delay is certainly smaller than the increased delay due to filling slot  $j$  using a flight in slot  $k$ . It follows that in the optimal solution  $x^*$  all the empty slots of the FPFS solution  $x^C$  must remain empty.

Part B) Hence a possible slot exchange may occur only between slots occupied in  $x^C$ . Let consider flight  $f$  in slot  $j$  and flight  $g$  in slot  $k$  two flights occupying slots in  $x^C$  where  $t_k^{CB} < t_j^{CB}$  and  $t_f$  and  $t_g$  are the sector entering times of flights  $f$  and  $g$ , respectively. In order to exchange slots  $k$  and  $j$ , it has to hold that  $ETO_f \leq t_g$  and  $ETO_g \leq t_j$ . But if

we exchange their slots and thus their entry times, the sum of their delays is constant. Hence  $x^C$  is optimal. ■

It follows that in the case of a single capacitated resource  $s$ , the FPFS approach minimizes the overall delay so maximizing the social welfare. However, it may not produce the best solution from airspace users' perspectives. In fact, there is a cost associated to the delay and this cost may vary depending on, e.g., the time of the day, the type of flight (connecting or not) and the type of airline [16]. The airspace users' community may wish to minimize the cost of the delays rather than delays.

If we assume that each flight has its own cost of one minute of delay and that its total cost of delay is proportional to the amount of delay, let us denote as  $w_f$  the cost of one minute of delay for flight  $f$ . The minimization of the aggregate cost of delay from the SR perspective is achieved through Model (2) where the cost of delay of each flight is considered in the objective function:

$$\min \sum_{f \in F} w_f \left( \sum_{t \in T} t x_{ft} - ETO_f \right) \quad (2a)$$

$$\sum_{f \in F} \sum_{t \in I_j} x_{ft} \leq 1 \quad \forall j \quad (2b)$$

$$\sum_{t \in T} t x_{ft} \geq ETO_f \quad \forall f \quad (2c)$$

$$\sum_{t \in T} x_{ft} = 1 \quad \forall f \quad (2d)$$

$$x_{ft} \in \{0, 1\} \quad \forall f \in F, t \in T \quad (2e)$$

A solution minimizing the total delay (i.e., optimal for Model (1)) does not necessarily minimize the total cost of delay (i.e., it is feasible but not necessarily optimal for Model (2)), and vice versa.

Model (2) obliges users to reveal their own costs of delay and to communicate them to the SR to perform the slot allocation. Any airline would be naturally encouraged to transmit a very large value of its own delay cost as to have its delay reduced as much as possible by the Model. If all airlines simultaneously display very large costs of delay we end up again to Model (1). Thus airlines would better meet their economic preferences if they all cooperate and communicate their true cost of delay. However since an opportunistic behavior cannot be prevented, the practical implementation of Model (2) is hard to achieve. An alternative approach would be to allow any airline to minimize its cost of delay, i.e., its delay, on its own without considering the other airlines' requirements, but this would lead to the original planned schedule which exceeds capacity. To overcome this deadlock we show that the introduction of a market based mechanism where airlines exchange slots at an appropriate price, allows to minimize the overall cost of delay without forcing airlines disclose private information.

## V. A MARKET-BASED MECHANISM

We propose a market-based mechanism to directly involve each airline in the decision making process of ATFM slot assignment. Such a system would let aircraft operators make independent decisions concerning delays incurred, while simultaneously guaranteeing the observance of capacity constraints (2b). A classical dual decomposition can be used to transform the centralized allocation problem (2) into a distributed market optimization. The resulting algorithm consists of two parts: local optimizations performed by the airlines that trade off the cost of deviating from the original schedule with the cost of purchasing network resources, and a central pricing mechanism that enforces constraint satisfaction by the airlines. In implementing the resulting market, the allocation solution obtained from the FPFS model is used as a baseline from which payment for resources is defined. Each airline needs only to purchase those resources above and beyond the ones received in the FPFS allocation, and may sell any resources which are not valued as highly as market prices dictate. Let us first introduce the following properties.

*Property 1:* Equations (2b)-(2d) can be reduced such that the remaining matrix is totally unimodular.

*Proof:* From (2c) it follows that if  $t < ETO_f$  then  $x_{ft} = 0$ . Then the remaining part of the equation, i.e.,

$$\sum_{t \in T | t \geq ETO_f} t x_{ft} \geq ETO_f \quad \forall f$$

is redundant with respect to constraints

$$\sum_{t \in T | t \geq ETO_f} x_{ft} = 1 \quad \forall f,$$

and thus can be eliminated.

If we assume that each slot is composed by only one minute, the matrix of constraints (2b) and (2d) is the assignment matrix that is totally unimodular. By adding a minute in one slot, we duplicate the appropriate columns in the matrix. But a matrix obtained by duplicating columns of a totally unimodular matrix is totally unimodular [17]. ■

*Property 2:* If a slot is empty in the solution  $x^c$  obtained by implementing the FPFS policy (i.e., the optimal solution of Model 1) it is not going to be used for minimizing the cost of the delay (Model 2).

*Proof:* By closing paralleling Part A of Theorem 1 ■ Hence constraints(2b) can be substituted with constraints

$$\sum_{f \in F} \sum_{t \in I_j} x_{ft} \leq \sum_{f \in F} \sum_{t \in I_j} x_{ft}^c \quad \forall j.$$

From Properties 1 and 2 it follows that the Model (2) with binary variables is equivalent to the following linear Model

(3):

$$Z_{SR} = \min \sum_{f \in F} w_f \left( \sum_{t \in T} t x_{ft} - ETO_f \right) \quad (3a)$$

$$- \sum_{f \in F} \sum_{t \in I_j} x_{ft} \geq - \sum_{f \in F} \sum_{t \in I_j} x_{ft}^C \quad \forall j \quad (3b)$$

$$\sum_{t \in T} x_{ft} = 1 \quad \forall f \quad (3c)$$

$$x_{ft} = 0 \quad \text{if } t < ETO_f \quad \forall f \in F, t \in T \quad (3d)$$

$$x_{ft} \geq 0 \quad \forall f \in F, t \in T \quad (3e)$$

Let us consider the nonnegative dual variables  $\lambda_j$  of constraints (3b). Economically the dual variable  $\lambda_j$  represents the price or value of slot  $j$ . For instance, its optimal value is zero when the demand for slot  $j$  is either equal to zero or to one. In both cases, slot  $j$  has no value. We define as  $\lambda$  the vector composed of all dual variables  $\lambda_j$ .

By relaxing constraints (3b), the corresponding lagrangian formulation of Model (3) is the following Model (4):

$$Z_L = \max_{\lambda} \min_x \sum_{f \in F} w_f \left( \sum_{t \in T} t x_{ft} - ETO_f \right) + \sum_j \lambda_j \left( \sum_{f \in F} \sum_{t \in I_j} (x_{ft} - x_{ft}^C) \right) \quad (4a)$$

$$\sum_{t \in T} x_{ft} = 1 \quad \forall f \quad (4b)$$

$$x_{ft} = 0 \quad \text{if } t < ETO_f \quad \forall f \in F, t \in T \quad (4c)$$

$$x_{ft} \geq 0 \quad \forall f \in F, t \in T \quad (4d)$$

The objective function (4a) is separable into  $|F|$  functions, one for each flight. Furthermore constraints (4b), (4c), and (4d) are also separable in term of flights, thus allowing the decomposition into  $|F|$  subproblems. Given  $\lambda_j$ , each airline has to solve the following problem for its flight  $f$ , in an independent way:

$$z_f(\lambda, x) = \min_x w_f \left( \sum_{t \in T} t x_{ft} - ETO_f \right) + \sum_j \lambda_j \sum_{t \in I_j} (x_{ft} - x_{ft}^C) \quad (5a)$$

$$\sum_{t \in T} x_{ft} = 1 \quad (5b)$$

$$x_{ft} = 0 \quad \text{if } t < ETO_f \quad t \in T \quad (5c)$$

$$x_{ft} \geq 0 \quad t \in T \quad (5d)$$

The solution obtained from the FPFS problem is taken as the baseline from which payments for resource utilizations are made. Thus a flight  $f$  which plans to enter sector  $s$  using slot  $j$  (i.e. for which  $x_{ft} = 1$ ,  $t \in I_j$ ) has to pay the correspondent price  $\lambda_j$ , unless it was already assigned the correspondent slot by the FPFS mechanism (i.e.  $x_{ft}^C = 1$ ,  $t \in I_j$ ). In the same way each slot  $k$  obtained from FPFS mechanism can be sold to another flight which values it more and receives the correspondent payment  $\lambda_k$  for it.

Once each flight has solved Model (5) it communicates to SR its slot request represented by the binary vector  $x_f = \{x_{f1}, \dots, x_{fT}\}$ . After receiving the solutions from all flights, SR checks for compatibility with a feasible slot allocation where at most one flight is allocated to each slot (constraints 3b). If no feasible solution is found SR updates the slot values  $\lambda_j$  according to a subgradient algorithm and communicates the updated values to the flights that in turn optimize again using Model (5). When SR reaches a feasible slot allocation, the algorithm stops.

The subgradient search algorithm iteratively updates prices seeking their optimal values. At iteration  $k + 1$  slot price  $\lambda_j^{k+1}$  is increased, with respect to the previous iteration, for an over-demanded slot and decreased for an unused one, always assigning a positive value, according to the following formula:

$$\lambda_j^{k+1} = \max(0, \lambda_j^k + S_r^k \cdot SG_j^k) \quad (6)$$

$$SG_j^k = \sum_{f \in F} \sum_{t \in I_j} x_{ft}^k - 1 \quad (7)$$

Where the step size  $S_r^k$  is determined according to a square summable but not summable rule because of its good convergence properties [18], i.e. satisfying

$$S_r^k \geq 0, \quad \sum_{k=1}^{\infty} (S_r^k)^2 < \infty, \quad \sum_{k=1}^{\infty} S_r^k = \infty \quad (8)$$

In this way the iterative auction is automatically implemented by solving Model (4) through the subgradient method.

The resulting market based distributed algorithm can be implemented as follows.

Market Algorithm:

- 1) SR sets prices  $\lambda_j^k$  to zero,
- 2) for  $k=1$  to MaxIterations do
- 3) SR transmits current prices  $\lambda_j^k$  to Airlines;
- 4) Airlines perform local allocation optimization with current prices  $\lambda_j^k$ , by solving Model (5);
- 5) Airlines transmit flight requests  $x_f$  to SR;
- 6) If constraints (3b) on slot capacity are respected then go to 11;
- 7) SR updates prices according to (6), (7) and (8);
- 8) end for
- 9) FPFS allocation of resources is implemented;
- 10) exit;
- 11) Market based allocation of resources is implemented;
- 12) exit.

When the Market Algorithm stops before the maximum number of allowed iterations is reached, it converges to a set of lagrangian multipliers that maximize Equation (4a) subject to constraints (4b) - (4d). Due to the complementary slackness conditions, the optimal value  $Z_L$  of the objective function

(4a) equals the optimal value  $Z_{SR}$  of the objective function (3a), and all constraints (3b) are satisfied. Thus Model (3) is solved and the overall cost of delay is minimized. In the optimal solution of the Market Algorithm a flight  $f$  may have exchanged its FPFS-allocated slot with a new one that was given up by another flight  $g \neq f$ . In turn flight  $g$  moves to a different slot previously occupied by flight  $h \neq g$ . Of course, this slot trade may occur if and only if all the flights involved find it economically profitable. For flight  $f$  the monetary profit between the FPFS and the Market slot allocation is equal to

$$p_f(\lambda, x) = w_f \left( \sum_{t \in T} t(x_{ft}^C - x_{ft}) \right) + \sum_j \lambda_j \sum_{t \in I_j} (x_{ft}^C - x_{ft})$$

where the first term is the difference of the cost of delay under the two allocations and the second one is the difference between the profit realized from the FPFS slot sale and the cost represented by the Market slot purchase.

*Property 3:* If  $\lambda$  is the set of optimal lagrangian multiplier and  $x$  is the optimal solution of the market algorithm, the monetary profit  $p_f(\lambda, x)$  is always non-negative for each flight  $f$ .

*Proof:* With little algebra, it holds that for any vector  $\lambda$ ,

$$z_f(\lambda, x) = w_f \left( \sum_{t \in T} tx_{ft}^c - ETO_f \right) - p_f(\lambda, x).$$

As  $x = x^C$  is a feasible solution for Model (5) and  $p_f(\lambda, x^C) = 0$ , the minimization of  $z_f(\lambda, x)$  implies that  $p_f(\lambda, x) \geq 0$ . ■

Property 3 implies that every actor finds it profitable to participate in the Market allocation of slots than to accept the FPFS solution. We also observe that multiple optimal dual solutions of Model (3) may exist. Thus the subgradient algorithm may converge to different sets of optimal lagrangian multipliers, depending on the initial step size for instance. It turns out that for some flight  $f$  the profit  $p_f(\lambda, x)$  (which is always non-negative thanks to Property 3) may not be constant when comparing the results of subgradient algorithms obtained with different step choices. However, due to the complementarity slackness conditions, the total optimal profit over all flights  $f \in F$  is constant and does not depend on the lagrangian multipliers, i.e.,

$$\sum_{f \in F} p_f(\lambda, x) = \sum_{f \in F} w_f \left( \sum_{t \in T} tx_{ft}^c - ETO_f \right).$$

Only the breakdown among flights may differ as a consequence of multiple optimal dual solutions of Model (3).

If the Market Algorithm does not converge within the maximum number of allowed iterations, the auction is not implemented and the FPFS solution is taken. However, our computational experiments based on real instances easily converge as described in the next Section.

## VI. EXAMPLE OF APPLICATION

To analyze the auction mechanism more in detail, consider the following examples based on real operational data.

### Case A: en-route regulation

This case is representative of the situation in which several flights are affected by the same and unique regulation limiting the maximum capacity on ATC en-route sector. This was the case, for instance, on the 2nd August 2008 between 04:00AM and 06:00AM, when a regulation was limiting the number of flights entering the French sector LFEERESMI to 14 per hour, for ATC capacity reasons. There were 18 flights originally planned to enter this sector at different points in time between 4:18AM and 5:51AM. The tactical cost of one minute of ground delay for each flight  $f$  (i.e.  $w_f$ ) was considered as a stochastic variable uniformly distributed between 5 €/min and 20 €/min, in line with the figures provided in [16]. From the baseline schedule obtained by applying FPFS, we simulated the interactive market mechanism, which converged to an optimal solution after 38 iterations. Table I contains the results of the simulation, where a cost to the company is indicated with a negative value, while an income with a positive one. Table II shows the different slots and their associated values.

TABLE I  
NUMERICAL RESULTS CASE A

Flight	$w_f$	$ETO_f$	FPFS slot	Market slot	Cost FPFS (Delay)	Cost Market (Delay+ slot trading)	$p_f$
F1	-16	04:18	04:18	04:18	0	0	0
F2	-17	04:24	04:24	04:24	0	0	0
F3	-8	04:25	04:25	04:25	0	0	0
F4	-6	04:26	04:30	04:30	-24	-24	0
F5	-6	04:36	04:36	04:36	0	0	0
F6	-14	04:44	04:44	04:44	0	0	0
F7	-9	04:45	04:47	05:17	-18	44	62
F8	-6	04:46	04:51	05:30	-30	98	128
F9	-19	04:47	04:55	04:47	-152	-87	65
F10	-10	04:48	05:00	05:08	-120	-88	32
F11	-16	04:53	05:04	04:53	-176	-136	40
F12	-13	04:54	05:08	04:55	-182	-173	9
F13	-17	05:00	05:12	05:00	-204	-160	44
F14	-15	05:04	05:17	05:04	-195	-131	64
F15	-7	05:12	05:21	05:21	-63	-63	0
F16	-11	05:24	05:25	05:25	-11	-11	0
F17	-18	05:37	05:37	05:37	0	0	0
F18	-8	05:51	05:51	05:51	0	0	0

TABLE II  
SLOT VALUES - CASE A

$I_j$	$\lambda_j$	$I_j$	$\lambda_j$
04:00 - 04:03	0	05:00 - 05:03	302.83
04:04 - 04:07	0	05:04 - 05:07	235.77
04:08 - 04:11	0	05:08 - 05:11	190.60
04:12 - 04:16	0	05:12 - 05:16	142.53
04:17 - 04:20	0	05:17 - 05:20	104.68
04:21 - 04:24	0	05:21 - 05:24	70.42
04:25 - 04:29	45.19	05:25 - 05:29	56.52
04:30 - 04:33	21.17	05:30 - 05:33	9.79
04:34 - 04:37	2.09	05:34 - 05:37	3.96
04:38 - 04:41	0	05:38 - 05:41	0
04:42 - 04:46	425.83	05:42 - 05:46	0
04:47 - 04:50	437.00	05:47 - 05:50	0
04:51 - 04:54	371.36	05:51 - 05:54	0
04:55 - 04:59	350.17	05:55 - 05:59	0

According to Property 3, the cost of delay caused by the FPFS system was greater or equal than the cost generated by the Market Algorithm (given by the cost of remaining delay + the cost of slot trading) for each flight. Under the FPFS policy, the overall delay is 91 minutes and the overall cost of delay is €1175. As expected, the Market Algorithm provides a higher overall delay (111 minutes), but a lower cost of delay (€863).

#### Case B: arrival airport regulation

This case is representative of the situation in which there is a limitation on the maximum number of flights arriving at an airport during a certain time period. We found that London City Airport (EGLC) was imposing a regulation on the 4th August 2008, limiting the arrival rate to 18 flight per hour, between 06:00AM and 07:30AM. This regulation was the only one affecting 24 flights, thus it was possible to simulate the use of our market mechanism to assign delays. The tactical cost of one minute of ground delay for each flight  $f$  (i.e.  $w_f$ ) was again considered uniformly distributed between 5 €/min and 20 €/min, in line with the figures provided in [16]. From the baseline schedule obtained by applying FPFS, we simulated the interactive market mechanism, which converged to an optimal solution after 56 iterations. Table III contains the results of the simulation. Table IV shows the different slots and their associated values.

TABLE III  
NUMERICAL RESULTS CASE B

Flight	$w_f$	$ETOs_f$	FPFS slot	Market slot	Cost FPFS (Delay)	Cost Market (Delay+ slot trading)	$p_f$
F1	-7	06:01	06:01	06:01	0	0	0
F2	-10	06:03	06:03	06:03	0	0	0
F3	-9	06:08	06:08	06:10	0	9	9
F4	-7	06:08	06:10	06:40	-14	-12	2
F5	-14	06:08	06:13	06:08	-70	36	106
F6	-16	06:15	06:16	06:15	-16	32	48
F7	-19	06:18	06:20	06:18	-38	-32	6
F8	-14	06:19	06:23	06:20	-56	-38	18
F9	-6	06:21	06:26	06:43	-30	27	57
F10	-19	06:22	06:30	06:23	-152	-101	51
F11	-11	06:22	06:33	06:26	-121	-114	7
F12	-20	06:28	06:36	06:30	-160	-105	55
F13	-10	06:30	06:40	06:36	-100	-93	7
F14	-12	06:33	06:43	06:33	-120	-89	31
F15	-15	06:46	06:46	06:46	0	0	0
F16	-18	06:55	06:55	06:55	0	0	0
F17	-14	06:55	06:56	06:56	-14	-14	0
F18	-11	07:00	07:00	07:00	0	0	0
F19	-10	07:00	07:03	07:03	-30	-30	0
F20	-17	07:09	07:09	07:09	0	0	0
F21	-9	07:09	07:10	07:10	-9	-9	0
F22	-13	07:12	07:13	07:13	-13	-13	0
F23	-14	07:15	07:16	07:16	-14	-14	0
F24	-8	07:23	07:23	07:23	0	0	0

According to Property 3, the cost of delay caused by the FPFS system was greater or equal than the cost generated by the Market Algorithm (given by the cost of remaining delay + the cost of slot trading) for each flight. Under the FPFS policy, the overall delay is 73 minutes and the overall cost of delay is

TABLE IV  
SLOT VALUES - CASE B

$I_j$	$\lambda_j$	$I_j$	$\lambda_j$
06:00 - 06:02	0	06:46 - 06:49	45.00
06:03 - 06:05	0	06:50 - 06:52	0
06:06 - 06:09	319.01	06:53 - 06:55	127.11
06:10 - 06:12	300.52	06:56 - 06:59	112.89
06:13 - 06:15	278.08	07:00 - 07:02	108.58
06:16 - 06:19	316.64	07:03 - 07:05	76.42
06:20 - 06:22	284.81	07:06 - 07:09	103.09
06:23 - 06:25	255.82	07:10 - 07:12	87.00
06:26 - 06:29	203.22	07:13 - 07:15	63.43
06:30 - 06:32	159.74	07:16 - 07:19	47.58
06:33 - 06:35	129.31	07:20 - 07:22	0
06:36 - 06:39	99.18	07:23 - 07:25	0
06:40 - 06:42	62.78	07:26 - 07:29	0
06:43 - 06:45	41.94	-	-

€957. As expected, the Market Algorithm provides a higher overall delay (77 minutes), but a lower cost of delay (€631).

## VII. CONCLUSIONS

The market mechanism proposed for the allocation of slots shows several characteristics that make it appealing. First it is decentralized, thus allowing the users to be actively involved in the slot assignment process, by expressing their preferences over different feasible solutions to the scheduling problem in the spirit of CDM. Additionally the computational workload introduced by the mechanism to find a solution is not a burden uniquely on the System Regulator which processes all data but is distributed among users. Such a mechanism constitutes an improvement to the monolithic central allocation employed today, as the FPFS model is taken as the baseline solution from which a better one is searched in the solution space. Every actor is better-off when a solution to the auction mechanism is achieved: high-valued flights can reduce their delays by acquiring resources while low-priority flights may receive reimburse for the resources they relinquish. There is the need of an exhaustive experimentation and validation on real data to guarantee the feasibility of implementation under different conditions and to tune the different parameters. Future research should consider non constant costs of delay, but of increasing magnitude as the delay increases following a piecewise linear function as suggested in [16]. The final target of this study would be the identification of a market-based policy for acquiring slots when flights are concerned with more than one regulated sectors, and different flights are subjects to regulations from different sectors. The use of combinatorial auctions seems to be a promising approach to deal with such a complex problem.

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## REFERENCES

- [1] A. Majumdar, "Understanding En-Route Sector Capacity in Europe", in *European Air Traffic Management: principles, practice and research*, A.J. Cook, Ashgate Ed., 2007, pp. 65-95.
- [2] SESAR J.U., SESAR Milestone Deliverable D3, "The ATM Target Concept", September 2007.
- [3] A.R. Odoni, "The flow management problem in air traffic control", in *Flow Control of Congested Networks*, A.R. Odoni, L. Bianco and G. Szego, Springer-Verlag Ed., 1987.
- [4] G. Andreatta and G. Romanin-Jacur, "Aircraft flow management under congestion", *Transportation Science*, vol. 21, No.4, pp. 249-253, 1987.
- [5] P. Vranas, D. Bertsimas, and A.R. Odoni, "The multi-airport ground-holding problem in air traffic control", *Operations Research*, vol.42, No.2, pp.249-261, 1994.
- [6] D. Bertsimas and S. Stock Patterson, "The air traffic flow management problem with enroute capacities", *Operations Research*, vol. 46, No.3, pp.406-422, 1998.
- [7] D. Bertsimas, G. Lulli, and A.R. Odoni, "The Air Traffic Flow Management Problem: An Integer Optimization Approach", in *Integer Programming and Combinatorial Optimization*, A. Lodi, A. Panconesi and G. Rinaldi, Springer-Verlag Ed., 2008.
- [8] R. Hoffman and M.O. Ball, "A comparison of formulations for the Single-Airport Ground Holding Problem with Banking Constraints", Technical Research Report, Institute for System Research - University of Maryland, 1997.
- [9] EUROCONTROL, "Airport CDM Operational Concept Document", v. 3.0, September 2006.
- [10] S.J. Rassenti, V.L. Smith and R.L. Bulfin, "A combinatorial auction mechanism for airport time slot allocation", *Bell Journal of Economics*, vol. 13, No. 2, pp. 402-417, 1982.
- [11] M.O. Ball, G.L. Donohue and K. Hoffman, "Auctions for the Safe, Efficient and Equitable Allocation of Airspace System Resources", in *Combinatorial Auctions*, P. Cramton, Y. Shoham and R. Steinberg, MIT Press Ed., 2006, pp. 507-538.
- [12] S.L. Waslander, "Multi-Agent System Design for Aerospace Applications", Ph.D. thesis, Stanford University, California, 2007.
- [13] M.O. Ball and G. Lulli, "Ground Delay Programs: optimizing over the included flight set based on distance", *Air Traffic Control Quarterly*, vol. 12, pp. 1-25, 2004.
- [14] EUROCONTROL CFMU, "Air Traffic Flow and Capacity Management Operations Users Manual", v. 12, September 2007.
- [15] EUROCONTROL PRC, "An Assessment of Air Traffic Management in Europe during the Calendar Year 2007 - Performance Review Report", May 2008.
- [16] A.J. Cook, G. Tanner, and S. Anderson, "Evaluating the True Cost to Airlines of One Minute of Airborne or Ground Delay", University of Westminster, May 2004.
- [17] G.L. Nemhauser and L.A. Wolsey, "Integer and Combinatorial Optimization", John Wiley & Sons Ed., 1999.
- [18] D.P. Bertsekas, "Nonlinear Programming", second edition, Athena Scientific Ed., 1999.

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