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Resource Allocation in Flow-Constrained Areas with Stochastic Termination Times Considering Both Optimistic and Pessimistic Reroutes

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***Abstract*— In this paper we address a stochastic air traffic flow management problem. Our problem arises when airspace congestion is predicted, usually because of a weather disturbance, so that the number of flights passing through a volume of airspace (flow constrained area – FCA) must be reduced. We formulate an optimization model for the assignment of dispositions to flights whose preferred flight plans pass through an FCA. For each flight, the disposition can be either to depart as scheduled but via a secondary route, or to use the originally intended route but to depart with a controlled (adjusted) departure time and accompanying ground delay. We model the possibility that the capacity of the FCA may increase at some future time once the weather activity clears. The model is a two-stage stochastic program that represents the time of this capacity windfall as a random variable, and determines expected costs given a second-stage decision, conditioning on that time. This paper extends our earlier work on this problem by allowing the initial reroutes to vary from pessimistic (initial trajectory avoids weather entirely) to optimistic (initial trajectory assumes weather not present). We conduct experiments allowing a range of such trajectories and draw conclusions regarding appropriate strategies.**

Keywords: ATM; Air Traffic Manegemnt; FCA; Flow Constraint Area; Rerouting; Stochastic Programing; Ground Delay; Airborne Delay.

I. Introduction

A flow-constrained area (FCA) is a region of the national airspace system (NAS) where a capacity-demand imbalance is expected, due to some unexpected condition such as adverse weather, security concerns, special-use airspace, or others. FCAs might be drawn as polygons in a two-dimensional space, although in practice they are usually represented by a single straight line, functioning as a cordon.

When an FCA has been defined, it is then often the case that an airspace flow program (AFP) is invoked by the Federal Aviation Administration (FAA). An AFP is a traffic management initiative (TMI) issued by the FAA to resolve the anticipated capacity-demand imbalance associated with the FCA. It is the goal of this paper to develop a method by which, given the aggregate data described here, specific orders for individual flights can be developed for a single FCA that a) maximize the utilization of the constrained airspace, b) prevent the capacity of the FCA from being exceeded, and c) achieve a system-wide delay minimization objective. We recognize that this model cannot be directly applied to AFP planning as it does not address issues related to the manner in which the FAA and the flight operators collaborate in reaching a final decision regarding each flight. Our goal here is to develop relevant stochastic optimization models. We intend to address issues related to collaborative decision making (CDM) in later papers.

II. Related Research

The research in this paper and our earlier work on this problem builds on stochastic ground holding models. Several stochastic integer programming models have been developed (1), (7), (8), (12). While our model of FCA capacity is conceptually similar to airport arrival capacity models, we also explicitly represent the possibility of reroutes, including their dynamic adjustment under stochastic changes in FCA capacity.

There is also a growing literature on airspace flow management problems. Our work also builds on earlier work by Nilim and his coauthors on the use of "hybrid" routes that hedge against airspace capacity changes. In (11), the rerouting of a single aircraft to avoid multiple storms and minimize the expected delay was examined. In this model, the weather uncertainty was treated as a two-state Markov chain, with the weather being stationary in location and either existing or not existing at each phase in time. A dynamic programming approach was used to solve the routing of the aircraft through a gridded airspace, and the aircraft was allowed to hedge by taking a path towards a storm with the possibility that the storm may resolve by the time the aircraft arrived. The focus of the work was on finding the optimal geometrical flight path of the aircraft, and not on allocation of time slots through the weather area. Follow-on work expanded to modeling multiple aircraft with multiple states of weather and attempted to consider capacity and separation constraints at the storms (10).

Initial steps at a concept of operations that describes the terminology, process, and technologies required to increase the effectiveness of uncertain weather information and the use of a probabilistic decision tree to model the state space of the weather scenarios was provided in (1). Making use of this framework is a model recently proposed that uses a decision-tree approach with two-stage stochastic linear programming with recourse to apportion flows of aircraft over multiple routing options in the presence of uncertain weather (4). In the model, an initial decision is made to assign flights to various paths to hedge against imperfect knowledge of weather conditions, and the decision is later revised using deterministic weather information at staging nodes on these network paths that are close enough to the weather that the upcoming weather activity is assumed known with perfect knowledge. Since this is a linear programming model, only continuous proportions of traffic flow can be obtained at an aggregate level, and not decisions on which individual flights should be sent and when they should arrive at the weather. In (8), a stochastic integer programming model is developed based on the use of scenario trees to addressed combined ground delay-rerouting strategies in response to en route weather events. While this model is conceptually more general than ours, by developing a more structured approach we hope to develop a more scalable model.

Recently, a Ration-by-Distance (RBD) method was proposed as an alternative to the Ration-by-Schedule (RBS) method currently used for Ground Delay Programs (GDPs) that maximizes expected throughput into an airport and minimizes total delay if the GDP cancels earlier than anticipated. This approach considers probabilities of scenarios of GDP cancellation times and assigns a greater proportion of delays to shorter-haul flights such that when the GDP clears and all flights are allowed to depart unrestricted, the aircraft are in such a position that the expected total delay can be minimized. While this problem was applied to GDPs, the principles of a probabilistic clearing time where there is a sudden increase in capacity and making initial decisions such that the aircraft are positioned to take the most advantage of the clearing is similar to our problem.

III. THE MODEL

A. Model Inputs

Our base model inputs consist of information about the FCA, which is consistent with the information used in AFP planning:

- Location of the FCA
- Nominal (good weather) capacity of the FCA
- Reduced (bad weather) capacity of the FCA
- Start time of the AFP
- Planned end time of the AFP

From a list of scheduled flights and their flight plans, we determine the set of flights whose paths cross the FCA and which therefore would be subject to departure time and/or route controls under an AFP. We also require a set of alternate routes for each flight. The alternate route for each flight should be dependent on the geometry of the FCA and the origin-destination pair it serves. These most likely would be submitted by carriers in response to an AFP; for the purposes of this paper it is assumed they are submitted exogenously, although for testing purposes it was necessary to synthesize some alternate routes.

B. Controls

In order not to exceed the (reduced) FCA capacity, each flight will be assigned one of two dispositions in the initial plan reacting to the FCA:

1. *The flight is assigned to its primary route, with a controlled departure time that is no earlier than its scheduled departure time.* Given an estimate of en route time, this is tantamount to an appointment (i.e., a slot) at the FCA boundary. Some flights might be important enough that they depart on time, the AFP notwithstanding. Other flights might be assigned some ground delay.
2. *The flight is assigned to its secondary route, and is assumed to depart at its scheduled departure time.* In contrast to our earlier work, in this paper we allow a much for flexible and general definition of secondary route. A "pessimistic" secondary route would employ a trajectory directed around the periphery of the FCA (this was the case considered in the earlier papers). At the extreme. an "optimistic" secondary route would fly directly at the FCA (even though the flight did not have appropriate slot). If the weather did not clear, such an optimistic route would turn away from the FCA as late as possible and they fly around the FCA periphery. Inspired by the work on Nilim and his coauthors, we also consider intermediate routes, which hedge between the optimistic and pessimistic ones.

Several assumptions underlie our model:

- We do not consider airborne holding as a metering mechanism to synchronize a flight on its primary route with its slot time at the FCA.
- We assume that any necessary number of flights can be assigned to their secondary routes without exceeding any capacity constraints in other parts of the airspace.
- We assume that, when the weather clears, the FCA capacity increases immediately ("in one step"), back to the nominal capacity.
- The random variable is the time at which the FCA capacity increases back to its nominal value. We assume that perfect knowledge of the realization of this random variable is not gained until the scenario actually occurs, and so no recourse can be taken until the scenario is realized.

C. Scenrios and future responses

The outputs of this model are:

1. An initial plan that designates whether a flight is assigned to its primary route or secondary route; for those assigned to their primary route an amount of ground delay (possibly zero)

is assigned. For those assigned to their secondary route a specific directional angle (possibly zero) is assigned.

2. A recourse action for each flight under each possible early clearance time.

We model the time at which the weather clears (i.e. FCA capacity increases) as a discrete random variable, with some exogenous distribution. For any realization of the capacity increase time, the flights in question will be in some particular configuration as specified in the initial plan. Some will have departed, either on their primary or secondary routes, some will already have completed their journeys, and some will still be at their departure airports.

Flights that were originally assigned to their primary route and that have already taken off will be assumed to continue with that plan. For any such flight, the primary route is assumed to be best, so no recourse action is necessary.

We now consider flights originally assigned to their primary route that have not yet taken off. We need not consider transferring them to their secondary routes, because if that were a good idea in the improved capacity situation, it would also have been a good idea in the initial plan. Thus, the only possible change in disposition for these flights involves potentially changing their controlled departure time, i.e. reducing their assigned ground delay.

All other flights not yet considered were originally assigned to their secondary routes, with departure times as originally scheduled. These secondary routes avoid the FCA somehow. Under the FCA capacity windfall, some of those flights may now have an opportunity to use the FCA. If a flight has not yet taken off, and it is decided that it can use the FCA, the lowest cost way to do this is to re-assign it back to their primary route, with some controlled departure time no earlier than their scheduled departure time. If, on the other hand, the flight has already taken off, then the only mechanism to allow it the use of the FCA is a hybrid route that includes that portion (and perhaps more) of the secondary route already flown, plus a deviation that traverses the FCA and presumably rejoins the primary route at some point after the FCA (see Figure 1). A flight that is already en route via its secondary route may or may not prefer such a hybrid path, depending on the difference in cost (time, fuel, etc.) between doing that and continuing on its secondary route. There may be many possible hybrid routes, and perhaps only a limited set of those would be reasonable.

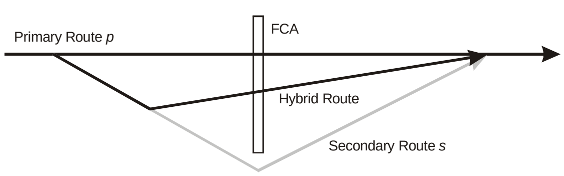


Figure 1 Reverting from secondary route back to primary route through FCA.

For each possible value of the capacity windfall time, we determine the expected locations of all affected flights at that time, and also what would be the best change in disposition, if any, for each of those flights according to a system performance metric. With this information, we can compute the conditional cost associated these flights adjusted based on the realization of the stochastic event. Ultimately, then, the goal of the optimization problem is to minimize the expected total cost, given these conditional costs and their probabilities.

D. Model Developments

We start by defining the discrete lattice on which time will be represented. We assume there is an index set $\{1,\ldots,T\}$ of size T that demarcates equally spaced time slots, each of duration Δt. Each of these represents a possible appointment time window at the FCA. The nominal capacity of the FCA should be specified in terms of the maximum number of flights permissible during one of these time windows. The number of time slots T then depends directly on Δt and the total duration of an AFP, perhaps inflated to allow for ending times later than the original estimate. The reference time $t=1$ can be chosen as the earliest scheduled departure time of all of the affected flights. The actual time indicated by the index t is then $(t-1/2)\Delta t$.

The flights affected by the FCA can be determined from the filed flight plans for that day, minus known cancellations and re-routes at the time the AFP is invoked. These flights are indexed according to the set $\{1,\ldots,F\}$. In the rest of the paper, any specific reference to a time period t and flight f assumes that $t\in\{1,2,\ldots,T\}$ and $f\in\{1,\ldots,F\}$.

1) Initial Plan

There are two sets of assignment variables that are related to decisions about the dispositions of flights. One set represents the initial plan, which is the set of decisions provided by the model that will be enacted immediately once the model is run and the AFP is declared. The second set represents conditional decisions (recourse actions) based on the random variable representing the time at which the capacity windfall takes place, which we do not know at the time of the execution of this optimization problem, but that we condition for when determining the best initial plan.

For the initial plan, we define the following set of binary decision variables:

$$x_{f,t}^{p}=\begin{cases}1, \text{ if flight } f \text{ uses its primary route and} \\ \quad \text{has an appointment time } t \text{ at the FCA} \\ 0, \text{ otherwise}\end{cases}$$

$$x_{f,r}^{s}=\begin{cases}1, \text{ if flight } f \text{ is assigned to its secondary route} \\ \text{that has a directional angle r} \\ 0, \text{ otherwise}\end{cases}$$

Every flight *f* needs to have an assigned disposition under the initial plan, thus:

$$\sum_t x_{f,t}^p + \sum_r x_{f,r}^s = 1 \quad \forall f \qquad (1)$$

We require that any flight that is assigned to its primary route cannot be given an appointment slot at the FCA that is earlier than its scheduled departure time plus the expected en route time required to arrive at the FCA. If $E_f \Delta t$ represents the en route time (from its origin to the FCA) for flight *f*, and $D_f \Delta t$ is the scheduled departure time for flight *f*, then:

$$\sum_{t=1}^{D_f + E_f} x_{f,t}^p = 0 \quad \forall f \qquad (2)$$

No similar constraint is applied to flights assigned to their secondary routes under the initial plan, because they are not metered at any point and hence are expected to depart at their originally scheduled departure time. There is no provision in the model for a flight to depart early, despite the fact that the secondary route takes more time than the primary route (since, subject to minor variations, airlines do not allow flights to take off before their scheduled departure times).

It might be the case that for a particular flight *f*, there is a latest slot time l_f at the FCA that the carrier who owns that flight would be willing to accept. Slots later than l_f can be prevented via the following constraint:

$$\sum_{t=l_f+1}^{T} x_{f,t}^p = 0 \qquad (3)$$

For any flight for which l_f is not explicitly provided, l_f is the time beyond which the secondary route will be chosen.

The initial constrained capacity (maximum number of flights) for time window *t* can now be defined as C_t^0 and the constraint to enforce it is:

$$\sum_f x_{f,t}^p \le C_t^0 \quad \forall t \qquad (4)$$

2) Second Stage

The variables and constraints defined so far represent the first stage of the stochastic program. It is assumed that these decisions will be enacted deterministically immediately after the FCA is declared. Next, we describe the second stage of the stochastic program – those variables that represent the conditional decisions we expect would be made if any of a number of possible capacity windfall times happens to come true in the future. We model the time slot at which this occurs as a discrete random variable with domain Ω and probability mass function

$$f_U(u) = \Pr\{U = u\} \quad \forall u \in \Omega$$

Under a capacity windfall, a flight that was originally assigned to its primary route with a controlled departure time might still be given the same general disposition, although its departure time could be moved earlier if that were beneficial to the system goal. We let

$$y_{f,t}^p \mid u = \begin{cases} 1, \text{ if at the time } U = u \text{ of the capacity windfall,} \\ \text{flight } f \text{ is assigned to its primary route with} \\ \text{appointment slot } t \text{ at the FCA} \\ 0, \text{ otherwise} \end{cases}$$

We will (shortly) introduce other variables for the other possible second stage flight dispositions, and we will require that all flights be assigned a disposition under every possible realization of the stochastic event *U*. For now, we proceed by obviating values of $y_{f,t}^p \mid u$ that would either be physically infeasible or politically imprudent. Later, structural constraints plus pressure from the objective function will lead to the best possible selection of second stage dispositions for all flights.

First, it is impossible to assign a flight to a slot that would require it to depart before its scheduled departure time:

$$y_{f,t}^p \mid u = x_{f,t}^p \quad \forall f, u, \quad \forall t \in \{1, \ldots, D_f + E_f\} \qquad (5)$$

This constraint works with constraint (2) to achieve the required result.

Given the timing *U* of the capacity windfall, some flights may already have taken off. If they did so via their primary route (with a controlled departure time), then their second stage disposition should match that of the first stage:

$$y_{f,t}^p \mid u = x_{f,t}^p \quad \forall f, u, \quad \forall t \in \{1, \ldots, u + E_f\} \qquad (6)$$

A closer look at constraint (6) reveals that it also satisfies an important requirement for flights that have not yet taken off. For any particular flight *f* and given the capacity windfall time *u*, the collection of primary stage variables $\left\{x_{f,t}^p\right\}_{t=1}^{t=u+E_f}$ will either contain one at exactly one position or it will consist entirely of zeros. In the former case, this means that the flight has already taken off, and that situation has been dealt with. In the latter case, this is indicative of the fact that these slot times are infeasible. Thus, even for flights that have not yet taken off, constraints (2) and (6) insure that they will not be assigned, in the second stage, to their primary routes with slot times that they cannot achieve.

Looking at constraints (5) and (6), it is clear that they can be combined:

$$y_{f,t}^p \mid u = x_{f,t}^p \quad \forall f, u, \quad \forall t \in \{1, \ldots, \max(u, D_f) + E_f\} \qquad (7)$$

On the other hand, for flights that already took off via their secondary routes (and therefore at their scheduled departure times), the only possible second stage dispositions are secondary or hybrid routes, so assignments to primary routes for these flights must be prevented:

$$\sum_t y^p_{f,t} \mid u \le 1 - \sum_r x^s_{f,r} \quad \forall u, \forall f \ni D_f < u \qquad (8)$$

In addition, we will not allow a flight whose controlled departure time is being moved in the face of a capacity windfall to be worse off than it was before this event materialized:

$$y^p_{f,t} \mid u \le \sum_{q \ge t} x^p_{f,q} + \sum_r x^s_{f,r} \quad \forall u, f, t \qquad (9)$$

Notice that we want to allow for the possibility that flights originally assigned to their secondary routes can revert, under the appropriate circumstances and if the optimization decides this is best, to their primary route if they have not already taken off, which is why the variable $x^s_{f,r}$ appears in constraint (9).

For flights that were originally assigned to the secondary route, the increased capacity at the FCA might allow some of these flights to pass through the FCA and thus improve their flight path by returning to the primary route at some point after the FCA or continuing directly to the destination. For a flight that has not yet departed, the same structure can apply, but the portions of the total flight path spent on the secondary and reverting routes have length zero. We define the second-stage decision variables for this choice as follows:

$$y^h_{f,t,r} \mid u = \begin{cases} 1, \text{ if flight } f \text{ was originally assigned to its} \\ \text{secondary route with directional angle r,} \\ \text{but under capacity clearing time } u \text{ has} \\ \text{been assigned an FCA appointment slot } t \\ 0, \text{ otherwise} \end{cases}$$

This decision can only be reached for flights that were originally assigned to their secondary routes:

$$y^h_{f,t} \mid u \le x^s_f \quad \forall u, f, t \qquad (10)$$

However, we note that the objective function will enforce this behavior implicitly. Such a flight will be on its secondary route, which may be altered to become a hybrid route that passes through the FCA. We need to impose constraints that insure that these flights are only assigned to FCA time slots they can feasibly reach. If a flight diverts from its secondary route to its hybrid route at time t^d there will be an earliest time it can reach the FCA. Figure 2 illustrates the geometry used to compute the parameter used by our model: $t^d_{f,t,r}$ is the time at which flight f must alter its secondary route to become a hybrid route that arrives at the FCA at time t. Figure 2 illustrates six different $t^d_{f,t,r}$ values, which depend on the initial secondary route, the clearance time and the associated geometry.

The following constraint prevents a flight from diverting to its hybrid route before the weather is actually cleared.

$$y^h_{f,t,r} \big| u = 0 \quad \forall f, r, u \quad \forall t \big| t^d_{f,t} \le u$$

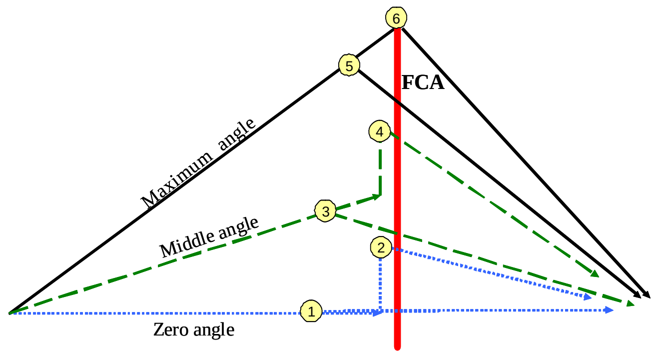


Figure 2 Possible conversions of a secondary into hybrid route.

In addition, the time slot assignment cannot be later than the latest time for which it would be reasonable to accept an assignment at the FCA considering the geometry of its secondary route:

$$y^h_{f,t} \mid u = 0 \quad \forall f, u, \quad \forall t > l_f$$

The final option possible is that a flight carries out its originally planned secondary route:

$$y^s_{f,r} \mid u = \begin{cases} 1, \text{ if flight } f \text{ was originally assigned to its} \\ \text{secondary route with directional angle r,} \\ \text{and if, under AFP stop time } u\text{, that} \\ \text{decision remains unchanged} \\ 0 \text{ otherwise} \end{cases}$$

Practically speaking, it would never make sense to assign a flight to its secondary route under the recourse if it had not also been given the same assignment in the initial plan. It might seem, therefore, that the following constraint is necessary:

$$y^s_f \mid u \le x^s_f \quad \forall u, f \qquad (11)$$

However, it can be seen that the objective function enforces this behavior implicitly. If it were cost-effective to assign a flight to its secondary route under the recourse, it would also be cost-effective to do so under the initial plan.

Constraints (10) and (11) can be combined into a single constraint:

$$\sum_t y^h_{f,t,r} \mid u + y^s_{f,r} \mid u \le x^s_{f,r} \quad \forall u, f, r$$

It would be possible, given the constraints developed so far, to assign a flight to a hybrid route that essentially reverts to the primary route immediately. In other words, this would be an assignment that is tantamount to taking off on the primary route at the scheduled departure time, which is a more logical way to interpret this outcome. Therefore we introduce the following constraint to enforce this behavior:

$$y^h_{f,D_f+E_f,r} \mid u = 0 \quad \forall f,u,r$$

For each time scenario *u*, every flight *f* must be assigned to one of these dispositions. Furthermore, if the disposition involves being scheduled into a slot appointment at the FCA, no more than one slot can be assigned to a given flight. Given that the decision variables are required to be binary, the following constraint addresses both of these concerns

$$\sum_t y^p_{f,t} \mid u + \sum_r \sum_t y^h_{f,t} \mid u + \sum_r y^s_f \mid u = 1 \quad \forall u, f \qquad (12)$$

For any value $U = u$, there will be a new capacity profile $C^u(t)$ that agrees with $C^0(t)$ up to time $t = u$, but represents an increase in capacity beyond that point. For example, if $C^0(t)$ had been a constant vector, then $C^u(t)$ could be a step function that makes a jump at time $t = u$. On the other hand, if $C^0(t)$ had been a periodic 0-1 function, then $C^u(t)$ might just have an increased duty cycle after time $t = u$. A wide variety of profiles for $C^u(t)$ are possible; the only real requirements are that it agree with $C^0(t)$ prior to time $t = u$, and that after that time, it supports a higher rate of flow than was possible under the initial plan. The capacity constraint under the scenario $U = u$ can now be written as:

$$\sum_f y^p_{f,t} \mid u + \sum_r \sum_f y^h_{f,t,r} \mid u \le C^u_t \quad \forall u,t \qquad (13)$$

3) Objective Function

Since our model involves the specification of decisions that are conditioned random events, the objective function will be an expected value. To emphasize the paradigm of creating a plan (our initial plan) together with contingency plans (our recourse actions), we represent the objective function as the sum of the deterministic cost of the initial plan minus the expected savings from recourse actions.

Therefore the objective function can thus be represented as:

$$Min\left[C(X) - \sum_u P_u S(Y_u)\right] \qquad (14)$$

Or more precisely:

$$Min \quad Z = z^1 + z^2 - \sum_u P_u \left(z^3_u + z^4_u\right) \qquad (15)$$

Where,

$$z^1 = \sum_f \sum_t c^p_{f,t} x^p_{f,t} \qquad (16)$$

$$z^2 = \sum_r \sum_f c^s_{f,r} x^s_{f,r} \qquad (17)$$

$$z^3_u = z^1 - \sum_f \sum_t c^p_{f,t} y^p_{f,t} \big| u + \sum_r \sum_f \sum_t c^s_{f,r} s^p_{f,t,r} \big| u \qquad (18)$$

$$z^4_u = \sum_r \sum_f \sum_t sv^h_{f,t,r} y^h_{f,t,r} \big| u \qquad (19)$$

where

$c^p_{f,t}$ is the cost of assigning flight *f* to its primary route so that it arrives at the FCA at time *t*.

$c^s_{f,r}$ is the cost of assigning flight *f* to its secondary route with directional angle r.

$sv^h_{f,t,r}$ is the savings incurred if flight *f* starts out on its secondary route with directional angle r but reverts to a hybrid route that arrives at the FCA at time *t*.

$s^p_{f,t,r}$ is a dummy binary variable that works as an indicator. It takes value of one when a flight initially assigned to its secondary route is assigned back to its primary route under revised plan.

So;

$$s^p_{f,t,r} = Min\left(x^s_{f,r}, y^p_{f,t,r}\right) \qquad (20)$$

IV. Computational Experiments

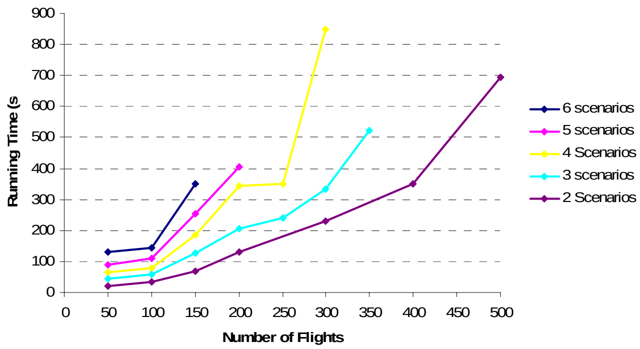
We conducted a set of computational experiments to investigate various properties of the model. The first set of experiments was aimed at evaluating the computational efficiency and scalability of the model. The second set sought to evaluate the ability of the model to improve decision making and the quality of air traffic operations. For both sets of experiments, flights, their routes, and alternate routes were generated artificially.

A. *Experiment1-Computational efficiency*

In the first set of experiments, several cases were considered with the lowest number of flights being 50 and the highest 500. Flight departure times were deterministically spaced evenly in time starting at 0 and ending at 100 minutes. We alternated among the three flight types. There were *T*=200 time slots; each slot had a width of Δt =2 minutes. Initially, the FCA had restricted capacity of 1 flight every two time slots (10 flights per hour). In all cases, the FCA cleared after 7 hours, at which time the capacity rose to infinity. There were 2, 3, 4, 5, or 6 possible early clearance times, each occurring with some positive probability; the sum of early clearance time probabilities equaled 0.95 so that the probability of no clearance was 0.05. In the event of early clearance, slot capacity rose from 1/2 to 2 flights for each time slot. For each number of scenarios (2, 3, 4, 5 or 6) the model was run for a different number of flights starting from 50 flights and increasing by 50 flights for each successive run until the maximum available computer memory was reached. A 2.8 GHz Intel® Pentium® based computer was used with

1.99 GB of RAM. The IP solver used was XPress MP® vers 2007B.

The following figures provide a presentation of the model's computational performance. They show the manner in which various problem parameters can ultimately limit the size of problem that can be solved. Note that the running time increases nonlinearly as a function of the number of flights with a greater rate of increase for larger numbers of scenarios. All cases were solved to optimality. The limiting factor was not the running time required to find an optimal solution but rather the memory required to initiate the solver. For example the maximum number of flights that could be handled with 6 scenarios (FCA clearance times) and 200 time slots is 150 flights. However we could solve a problem optimally with 500 flights, 200 time slots and 2 scenarios. (The number of time slots is 200 for the graphs below; however if the number of time slots is reduced to 150 then the model can be run either for larger numbers of flights or for more scenarios. Some of the corresponding results are shown in the table (1).)



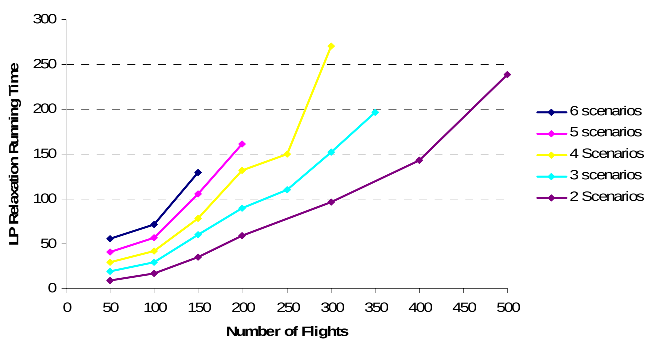


Figure 1. (a)Total running time (sec)vs. number of flights
(b) LP relaxation running time (sec) vs number of flights

B. *Experiment2-Decision Impacts*

To evaluate the decision impact of our model we ran a set of experiments, where we varied the scope of the decision space of the model. In this way we were able to mimic alternate decision support environments where less powerful models or operational options were available. The table below lays out the possible options in the planning and execution of the traffic flow management initiative. The cases vary relative to the extent to which recourse actions are allowed and planned for. A recourse action is taken if the weather clears earlier than expected. In the ground delay case, this means a flight is released at a time earlier than its planned departure time. In the reroute case, this means a flight adjusts its original planned reroute to a more direct route. The key novel contribution of our model is its ability to take into account recourse actions when generating its initial plan. In the table of options below note that the manner in which recourse is handled can vary from the planning to execution steps. In fact, while in many operational contexts recourse actions are taken during execution, it is rarely the case that the initial plan is made anticipating the possibility of recourse actions.

	Reroute (RR)	**Ground Delay (GD)**
Plan	*none/static/recourse*	*none/static/recourse*
Execute	*none/static/recourse*	*none/static/recourse*

The following alternate cases were considered.

Case 1:	**Plan**	**RR:** *none*	**GD:** *recourse*
	Execute	**RR:** *none*	**GD:** *recourse*

This case eliminates totally the reroute option but uses the full power of the model in planning and executing the ground delay plan. This is perhaps not a realistic case, but by comparing it to Case 2, we can isolate the value of recourse in ground delay planning.

Case 2:	**Plan**	**RR:** *static*	**GD:** *static*
	Execute	**RR:** *static*	**GD:** *static*

This case chooses the best single static plan and then sticks with that plan during execution even if the weather clears early. In terms of practice this is probably an overly pessimistic scenario since usually there is some recourse in the execution step.

Case 3:	**Plan**	**RR:** *static*	**GD:** *static*
	Execute	**RR:** *static*	**GD:** *recourse*

This case finds the best static plan but only allows recourse in the ground delay execution. This is a plausible representation of reality.

Case 4:	**Plan**	**RR:** *static*	**GD:** *recourse*
	Execute	**RR:** *static*	**GD:** *recourse*

This case allows full recourse ground delay planning and execution but static reroute planning and execution. This is another realistic scenario under which TFM execution systems are not responsive enough to provide dynamic rerouting for en route flights.

Case 5:	**Plan**	**RR:** *static*	**GD:** *recourse*
	Execute	**RR:** *recourse*	**GD:** *recourse*

This case allows full recourse ground delay planning and execution but static reroute planning with recourse reroute execution.

Case 6:	Plan	RR: *static*	GD: *static*
	Execute	RR: *recourse*	GD: *recourse*

This case chooses the best single static plan but then allows recourse actions when the plan is executed. This is another plausible representation of reality, although a fairly optimistic one, in that the static plan is optimized and it is assumed that each flight is able to take the best recourse action during execution.

Case 7:	Plan	RR: *recourse*	GD: *recourse*
	Execute	RR: *recourse*	GD: *recourse*

This case applies the full power of the model.

Case 8-13:	Plan	RR: *recourse*	GD: *recourse*
	Execute	RR: *recourse*	GD: *recourse*

Case 8-13 applies the full power of the new model, structured to consider and evaluate alternate secondary route options sets.

The tables below provide the results of an experiment under which all 13 cases were executed. Cases 1 through 7 use only the pessimistic reroute option as defined earlier. Among these, Case 7 applies the most powerful combination of planning and execution and thus generates the lowest cost solution. Perhaps not surprisingly, there is a very substantial cost difference in all experiments between Cases 2 and 7, which compare a totally static system with a totally dynamic one. What is perhaps more surprising is that substantial savings are still obtained when one compares Case 7 to Cases 3, 4, 5 and 6. Of particular note is that substantial savings still remain even when comparing Cases 7 and 6. Case 6 is a very optimistic representation of a TFM system that uses optimized static planning but then has the full (optimized) power of recourse during execution. Even in this case, the model can achieve savings in the range of 10 to 20%. It is also interesting to compare Cases 3 and 4. The only difference between these cases is that Case 4 takes recourse into account in making its ground delay decisions. It is noteworthy that this produces a very substantial impact – in some cases, savings of over 25%. Of course, we have already noted that planning with recourse in the context of ground delay programs is already practiced through the application of exemption policies, e.g. using distance-based GDP's (2). GDP recourse planning is fully analyzed in (5). The importance of the model developed in this paper is that it can simultaneously carry out recourse-based ground delay and reroute planning.

In case 8, we allow the directional angle of the secondary route for each flight to be chosen from two options. a) zero-angle (this is the "optimistic" case -- the secondary route has no angular deviation from its primary route); b) Max-angle (this is the "pessimistic" case – the trajectory for the secondary route totally avoids the FCA). It is interesting to note that substantial savings are achieved by case 8 relative to case 7. This shows that the strategy of flying towards a weather impacted area anticipating future clearance can produce the lowest expected cost. What is more important to note is that our model can determine when this represent an optimal strategy.

In case 9 we add a third option with a directional angle equal to the average of the optimistic and pessimistic cases (the average of the zero-angle and the max angle). Our results show that this produces very little additional benefit.

For case 10, we pre-compute the optimal directional angle of each flight independent of capacity constraints (here optimal is relative to the weather clearance time probability distribution and the geometry of the flight path). Note that the costs here are worse than in case 8 indicating the importance of considering the capacity constraints.

Cases 11, 12 and 13 are similar to cases 8, 9 and 10 respectively except that the capacity of the second stage was set to infinity (un-capacitated). In the absence of second stage capacity constraints, as one can expect, the approach that employs the optimal unconstrained directional angle (case 13) provides the lowest cost. *However, a very significant result is that the differences between case 13 and cases 11 and 12 are negligible. Thus, this experiment suggests that it is not necessary to consider reroutes that hedge against the extremes. Rather one can achieve nearly all the benefits by choosing one extreme or the other.*

The second and the third columns are the clearance time and its probability. The fourth and the fifth columns are the total costs for ground delays and airborne delays of the first stage. The sixth and the seventh columns are the savings occurred to the first stage costs (recourses) due to increased capacity on the second stage. The eighth column is the total cost of the system for each realization of the random variable (Clearance time). The last column is the objective function value which is the minimum expected total cost. The units of all costs are "number of time slots" that can be transferred to minutes or dollars.

V. Conclusion and Future work

In this paper, we have defined the basics of a stochastic optimization model for simultaneously making ground delay and reroute decisions in response to en route airspace congestion. We have also given the results of computational experiments that both test the computational efficiency and decision impact of the model. These results show that the model is tractable and can serve as a basis for solving practical TFM problems using commercial IP solvers. Further, the results show that the models have the potential to substantially improve TFM decision making.

While our model is able to produce better dynamic, stochastic TFM plans, it will only be useful if TFM systems can dynamically adjust to changing conditions. Today TFM systems in the U.S. dynamically adjust ground delay decisions, e.g. by allowing flights given ground delays to leave early if the weather clears; however, there is less ability to dynamically reroute airborne flights to take advantage of newly availability capacity. It is also worth noting that such dynamically adaptive systems have less predictability that more static ones. In that sense, the use of models such as ours requires the user to make certain tradeoffs between expected delay and predictability.

Our model can be re-run if, and as often as, real-time information suggest that the data supporting a previous execution of the model have changed significantly, for example, if carriers cancel some additional flights, or if the probabilistic weather forecast changes. The model can be forced to preserve earlier decisions by additional constraints fixing those decisions for flights currently in the air.

We anticipate the need to provide many refinements and extensions to this model to better address practical problem solving. Further, another vital direction is the development of strategies necessary to embed this model within CDM processes necessary for the delivery of practical air traffic flow management solutions.

TABLE 2 Experiment 1

Case	q	p[q]	c(xp=1)	c(xs=1)	sv(yp=1)	sv(yh=1)	c(q)	Objective
	15	0.5			11079	0	1742	
1	30	0.3	12821	0	9511	0	3310	3619
	45	0.1			8096	0	4725	
	15	0.5			0	0	4686	
2	30	0.3	355	1444	0	0	4686	4686
	45	0.1			0	0	4686	
	15	0.5			56	0	4630	
3	30	0.3	355	1444	0	0	4686	4658
	45	0.1			0	0	4686	
	15	0.5			7764	0	2113	
4	30	0.3	8805	357	6675	0	3202	3423
	45	0.1			5692	0	4184	
	15	0.5			7652	298	1330	
5	30	0.3	8805	357	6570	216	2659	2837
	45	0.1			5537	158	3867	
	15	0.5			56	1035	1526	
6	30	0.3	355	1444	0	690	2615	2373
	45	0.1			0	371	3573	
	15	0.5			1169	1104	1025	
7	30	0.3	1551	1318	914	829	2105	2021
	45	0.1			728	504	3264	
	15	0.5			690	2340	519	
8	30	0.3	927	2433	522	2207	1084	1676
	45	0.1			398	1712	2692	
	15	0.5			566	2273	543	
9	30	0.3	788	2380	425	2162	1017	1638
	45	0.1			300	1647	2686	
	15	0.5			839	3230	490	
10	30	0.3	1162	3286	594	3174	903	1834
	45	0.1			424	2812	2159	
	15	0.5			360	2602	253	
11	30	0.3	455	2655	300	2484	669	1342
	45	0.1			240	2152	1723	
	15	0.5			360	2562	265	
12	30	0.3	455	2618	300	2443	681	1341
	45	0.1			240	2111	1735	
	15	0.5			906	3561	195	
13	30	0.3	1075	3570	751	3551	381	1340
	45	0.1			601	3404	971	

Experiment 1 note: reduced capacity = 1 flight every 4 minutes; increased capacity = 1 flights every 1 minutes; airborne delay cost/ground delay cost: a = 3; number of flights=160.

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