

A Multi-stakeholder Evaluation of Strategic Slot Allocation Schemes under Airline Frequency Competition

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Abstract—In addition to the increasing passenger demand, airline frequency competition is another reason for the growing demand for airport resources. By providing more flight frequency, an airline attracts more passengers. As a result, demand for flight operations often exceeds capacity at congested airports, resulting in delays and disruptions. At some congested airports, the limited airport capacity is allocated between different airlines using administrative slot controls. At a slot controlled airport, (a) the total number of allocated slots, and (b) the distribution of slots across different airlines, together determine the effectiveness of any slot control strategy. We propose a game-theoretic model of airline frequency competition under administrative slot controls. The model is based on the popular S-shaped relationship between market share and frequency share of an airline. The model is solved for a Nash equilibrium and the model predictions are validated against actual frequency data, with the results indicating a good fit. We describe two different schemes for distributing the available slots among different airlines. We evaluate the impact of varying the total number of allocated slots on the airlines and the passengers. The results from a case-study at the New York LaGuardia (LGA) airport suggest that a small reduction in the total number of allocated slots translates into a substantial reduction in flight and passenger delays, and a considerable improvement in airlines' profits.

Keywords—airline frequency competition; airline scheduling; airport slots; flight delays; administrative slot controls; S-curve; Nash equilibrium; game theory; slot reduction.

I. INTRODUCTION

Growing congestion at major US airports leads to several billion dollars of delay costs each year. Total flight delays rose sharply during much of the early 2000s. Although the current economic recession has led to airline schedule reductions and consequently resulted in delay reduction over the last couple of years, large delays are expected to return as soon as the economic crisis subsides [1]. Aircraft delays result in passenger delays and discomfort, as well as additional fuel consumption and green house gas emissions. Various studies have estimated the total economic impacts of delays. For the calendar year 2007, which was the last full year of peak air travel demand before the economic downturn, the total cost of U.S. air transportation delays was estimated at \$33.2 billion [2]. The magnitude of these delays can be properly grasped by noting that during the same year, the aggregate profits of US domestic airlines were \$5.0 billion [3].

A. Demand-Capacity Mismatch

According to the Bureau of Transportation Statistics (BTS), around 50% of the delayed flights during the year 2007 were categorized as flights delayed due to the National Aviation System (NAS) [4]. Weather and volume were the top two causes of NAS delays. Delays due to volume are those caused due to scheduling more airport operations than the available capacity, while the delays due to weather are those caused by airport capacity reductions under adverse weather conditions. Both these types of delays are due to scheduling more operations than the realized capacity. Such mismatches between demand and capacity are a primary cause of flight delays in the United States.

Increasing capacity and decreasing demand are the two natural ways of bringing the demand-capacity mismatch into balance. Capacity enhancement measures such as building new airports, construction of new runways, etc. are investment intensive, require long-time horizons, and might not be feasible in many cases due to geographic, environmental, socio-economic and political issues associated with such large projects. On the other hand, demand management strategies have the potential to restore the demand-capacity balance over a medium- to short-time horizon with comparatively little investment. Demand management strategies refer to any administrative or economic policies and regulations that restrict airport access to users. All the demand management strategies proposed in the literature and practiced in reality can be broadly categorized as administrative controls and market-based mechanisms, although various hybrid schemes have also been proposed. This paper focuses specifically on administrative slot control strategies.

B. Administrative Slot Controls

Over the recent years, some of the most congested airports in the United States, including LaGuardia, John F. Kennedy, and Newark airports in the New York region, Reagan airport at Washington D.C., and O'Hare airport at Chicago have had some form of administrative controls limiting the number of flight operations. Outside of the US, administrative controls are commonplace at busy airports. Several major airports in Europe and Asia are 'schedule-coordinated', where a central coordinator allocates the airport slots to airlines based on a set of pre-determined rules. Under the current practices, both in and outside of the US, the available slots are allocated among different carriers according to criteria based on historical

precedents and use-it-or-lose-it rules. Under these rules, an airline is entitled to retain a slot that was allocated to it in the previous year, contingent on the fact that the slot was utilized for at least a certain minimum fraction of time over the previous year. An airline failing to utilize a slot frequently enough, however, is in danger of losing it.

Implicit in the current approach for setting the slot controls is the need to make a tradeoff between delays and resource utilization. Specifically, it requires ascertaining the 'declared' capacity of an airport beforehand even though the actual capacity on the day of operations is a function of prevalent weather conditions. Declaring too large a value for capacity poses the danger of large delays under bad weather situations and declaring too low a value leads to wastage of resources under good weather conditions. Declared capacity, that is, the total number of slots to be allocated per time period, greatly affects the congestion and delays at an airport.

In order to determine the most appropriate value of declared capacity, it is very important to distinguish between the demand for airport capacity in terms of the number of flight operations and the passenger demand for air travel in terms of number of enplanements. It is the former that affects the airport congestion most directly. Table I shows the values of total number of passengers, total number of flights and total arrival delays to flights in the US. All the values in Table I are normalized such that the values for the year 2000 are all equal to 100. Passenger demand dipped in the first two years of this decade following the economic recession and the 9/11 attacks. However, the period from 2002 to 2007 saw a sustained growth in passenger demand. By 2007, the passenger demand was 13.28% higher compared to that in 2000. However, the number of scheduled flight operations was 24.46% higher and total arrival delays to flights were 38.58% higher.

Such disproportionate rise in number of flight operations compared to a relatively moderate increase in number of passengers meant that the average number of passengers per flight reduced by around 9% from 2000 to 2007. This suggests that there is more to the demand-capacity mismatch than simply the rate of passenger growth outpacing the rate of airport capacity expansion. By providing more frequency of flights, an airline attracts more passengers. Frequency competition between carriers is considered partially responsible for exacerbating the demand-capacity mismatch and therefore the congestion problem.

In this paper, we model airline frequency decisions under competition and evaluate the impact of two strategic administrative slot control schemes on various performance metrics from the viewpoints of airlines and passengers.

II. BACKGROUND

Frequency planning is that part of the airline schedule development process which involves decisions about the number of flights to be operated on each route. Given an estimate of total demand on a route, the market share of each airline depends on its own frequency as well as on the competitor frequency. Market share can be modeled according to the so-called S-curve or sigmoidal relationship between the market share and frequency share, which is a widely accepted

notion in the airline industry ([5], [6]). Empirical evidence of the relationship was documented in some early studies and regression analysis was used to estimate the model parameters ([7], [8], [9]). Over the years, there have been several references to the S-curve including [10] and [11]. In a recent study, [12] provided further statistical support for the S-curve, based on a nested Logit model for nonstop duopoly markets. The most commonly used mathematical expression for the S-curve ([6], [9]) is given by,

$$MS_i = \frac{FS_i^\alpha}{\sum_{j=1}^n FS_j^\alpha}, \quad (1)$$

where MS_i is the market share of airline i , FS_i is the frequency share of airline i , n is the number of competing airlines, and α is a model parameter.

Game-theoretic literature capturing the S-curve-based frequency competition is limited. Most of the previous studies involving game theoretic analysis of frequency competition, such as [13], [14], [15], [16], [17], [18] and [19], model market share using Logit- or nested Logit-type models, with utility typically being an affine function of the inverse of frequency. Depending on the exact values of the utility parameters, such relationships can be considerably different from the S-shaped relationship between market share and frequency share. In this research, we use one of the most popular characterizations of the S-curve model. Reference [18] modeled schedule and fare competition as a strategic form game for a sample problem comprising six airports and two airlines. Reference [13] modeled airline competition on fare, frequency and aircraft sizes as an extensive form game and presented equilibrium results for a network comprising four airports and two airlines. Neither of these studies provides any empirical justification of suitability of Nash equilibrium outcome. Reference [15] analyzed frequency competition in a hub-dominated environment using a strategic form game model and presented results for a large network of realistic size involving multiple airlines. This study reported significant disparities between the model predictions and the state of the actual system. Each of these three studies adopted a successive optimization approach to solve for a Nash equilibrium. In this paper, we also use a successive optimizations approach for the computation of a Nash equilibrium and provide empirical validation of our equilibrium predictions.

In most of the previous research, scheduling decisions on one segment are not constrained by the schedule on other segments. (We define a segment as an origin and destination pair for nonstop flights.) This is a good approximation for a situation where an airport is not congested, and takeoff and landing slots are freely available. But some congested US airports and several major airports in Europe and Asia are slot controlled. With projected demand in the US expected to outpace the expansion of airport capacity, there is a possibility of many more airports in the US employing some form of demand management in the future. At a slot controlled airport, increasing the frequency of flights on one segment usually requires the airline to decrease the frequency on some other segment from that airport. To the best of the authors' knowledge, no previous study has incorporated such slot constraints into airline competition models.

TABLE I. TREND IN NUMBER OF PASSENGERS, FLIGHTS AND DELAYS

Year	Number of Passengers	Number of Flights	Total Arrival Delays to Flights (Minutes)
2000	100.00	100.00	100.00
2001	93.34	96.47	78.15
2002	92.06	102.32	59.75
2003	97.29	119.65	75.18
2004	105.04	126.09	103.58
2005	109.62	126.98	107.80
2006	109.81	122.86	120.99
2007	113.28	124.46	138.58
2008	108.70	118.60	119.11
2009	103.07	110.73	91.82

The main contributions of this paper are threefold. First, we propose a model of airline frequency competition which is (a) compatible with the S-curve relationship between market share and frequency share, and (b) captures airport slot constraints. Second, we provide empirical validation of our model using actual frequency data. Finally, using this model, we evaluate two different administrative slot allocation schemes from the perspectives of airlines and passengers. The rest of the paper is organized as follows. Section III describes the game theoretic model of airline frequency competition. Section IV provides a detailed description of the data used for the numerical experiments and empirical validation results. Section V describes two different schemes for distributing the available slots among different airlines. Section VI describes the numerical experiments and results. Finally, section VII concludes the paper with a summary and discussion of the main results.

III. GAME THEORETIC MODEL

This section explains the relevant mathematical notations and describes the model. In sub-section A, we describe the basic model of airline frequency competition based on the S-curve as an optimization problem for an airline. In sub-sections B and C of this section, we present two important extensions of this model. Sub-section D briefly discusses the solution algorithm.

A. Basic Model

Consider an airline operating at a slot-controlled airport. A slot available to an airline can be used for a flight to or from any other airport, but the total number of slots available to each airline is limited. We will consider only the flight departures from a slot controlled airport and assume that the airports at the other end are not slot controlled. This assumption is quite reasonable in the US context, where only a handful of airports are slot constrained. We focus only on the daily allocation of slots while ignoring the time-of-the-day aspects.

To begin with, we consider frequency planning decisions while assuming that the aircraft sizes remain constant for each segment. We will analyze the impact of this assumption later, in sub-section D of section VI, by partially relaxing this

assumption. We propose a multi-player model of frequency competition where each airline is a player and each airline's decision problem is represented as an optimization problem. From here onwards, this model will be referred to as the *basic model*. In this *basic model*, the only decision variables are the numbers of nonstop flights of the airline on each segment with origin at the slot controlled airport. This *basic model* is applicable for situations where the fares and other factors are similar among the competing airlines and the main differentiating factor between different airlines is the service frequency.

Let S_a be the set of potential segments for airline a with origin at the slot controlled airport. Let p_{as} be the average fare charged by airline a on segment s . Let Q_{as} be the number of passengers carried by airline a on segment s . In general, a passenger might travel on more than one segment to go from his origin to destination, which in some cases involves connecting between flights at an intermediate airport. However, we will assume segment-based demand, that is, a passenger traveling on two different segments will be considered as a part of the demand on each segment. This assumption is quite reasonable for the airports in New York city area where nearly 75% of the passengers are nonstop [20], but not very accurate for major transfer hubs such as the Chicago O'Hare airport. Let the total passenger demand on segment s be M_s . C_{as} is the operating cost per flight for airline a on segment s . S_{as} is the seating capacity of each flight of airline a on segment s . Let α_s be the exponent in the S-shaped relationship between the market share and the frequency share on the nonstop segment s . The value of α_s depends on the market's characteristics such as long-haul/short-haul, proportion of business/leisure passengers, etc.

The vector of decision variables for airline a is $[f_{as}]_{s \in S_a}$. Because the origin airport is slot constrained, the maximum number of flights that can be scheduled by airline a is restricted to U_a . Often, under the current set of administrative policies based on use-it-or-lose-it type rules, there are restrictions on the minimum number of slots that must be utilized by an airline in order to avoid losing slots for the next year. So there may be a lower limit on the number of slots that must be used. Let L_a be the minimum number of slots that must be utilized by airline a . Let \mathbb{A} be the set of all airlines and let \mathbb{A}_s be the set of airlines operating flights on segment s .

As defined by the S-curve relationship, the market share of airline a on nonstop segment s is $(f_{as}^{\alpha_s}) / (\sum_{a' \in \mathbb{A}_s} f_{a's}^{\alpha_s})$, which provides an upper bound on the number of passengers for a specific carrier on a specific segment. This restriction is imposed by constraint (3) in the model that follows. Obviously, the number of passengers on a segment cannot exceed the number of seats. Moreover, due to demand uncertainty and due to the effects of revenue management, the airlines are rarely able to sell all the seats on an aircraft. Assuming a maximum average segment load factor of LF_{max} , the seating capacity restriction is modeled by constraint (4). We present results assuming 85% as the maximum average segment load factor value. We test the sensitivity of the results to variations in this value later in sub-section C of section VI. The objective function (2) to be maximized is the total

operating profit, which is total fare revenue minus total flight operating cost. The overall optimization model is as follows,

$$\text{maximize} \quad \sum_{s \in \mathbb{S}_a} p_{as} Q_{as} - C_{as} f_{as} \quad (2)$$

$$\text{subject to:} \quad Q_{as} \leq \frac{f_{as}^{\alpha_s}}{\sum_{a' \in \mathbb{A}_s} f_{a's}^{\alpha_s}} M_s \quad \forall s \in \mathbb{S}_a \quad (3)$$

$$Q_{as} \leq LF_{\max} S_{as} f_{as} \quad \forall s \in \mathbb{S}_a \quad (4)$$

$$\sum_{s \in \mathbb{S}_a} f_{as} \leq U_a \quad (5)$$

$$\sum_{s \in \mathbb{S}_a} f_{as} \geq L_a \quad (6)$$

$$f_{as} \in \mathbb{Z}^+ \quad \forall s \in \mathbb{S}_a \quad (7)$$

The market share of each airline depends on the frequency of other competing airlines in the same market, which in turn are decision variables of those other airlines. Therefore, this is multi-agent model. The optimization problem given by (2) through (7) can only be solved for a given set of values of competitors' frequencies.

We now propose two extensions to the *basic model*. The first extension is applicable to segments where the competing carriers differ in terms of fare charged or in some other important way. The second extension is applicable to segments on which only one carrier currently operates nonstop flights.

B. Extension I: Fare Differentiation

The *basic model* assumes that the market share on each segment depends solely on the frequency share on that segment. This assumption is reasonable in many markets where the competitor fares are very close to each other and the competing airlines are similar from the perspectives of the passengers in other ways. However, for markets where the fares are different, the basic S-curve relationship can be a poor approximation of actual market shares. Consider a market where the competing airlines are differentiated in both fare and frequency. Different types of the passengers would react differently to these attributes. While some passengers value lower fares more, others give more importance to higher frequency and the associated greater flexibility in scheduling their travel. In addition, there could be other airline-specific factors that impact the passenger share. For example, some passengers might have a preference for the big legacy carriers operating wide-body or narrow-body fleets over the regional carriers operating turbo-prop aircraft or small regional jets. To incorporate these effects, we propose an extension of inequality (3). Let there be T types of passengers. Let γ_s^t be the fraction of segment s passengers belonging to type t , such that $\sum_{t=1}^T \gamma_s^t = 1$. Let α_s^t be the frequency elasticity of type t passengers, which serves the same purpose as the exponent of the S-curve in the *basic model*. Let β_s^t be the fare elasticity of type t passengers. Obviously, we expect α_s^t to be non-negative and β_s^t to be non-positive. Let θ_a be the airline specific factor for airline a . Inequality (3) can then be extended as,

$$Q_{as} \leq \sum_{t=1}^T \frac{\theta_a f_{as}^{\alpha_s^t} p_{as}^{\beta_s^t}}{\sum_{a' \in \mathbb{A}_s} \theta_{a'} f_{a's}^{\alpha_s^t} p_{a's}^{\beta_s^t}} \gamma_s^t M_s \quad (8)$$

The market share of each airline is now a function the fares, frequencies, and airline specific factors of all competing airlines. This model incorporates the effects of different fares and frequencies on the passenger shares. Also, it can model multiple passenger types such as leisure vs. business, by specifying different fare and schedule elasticities for different type of passengers. Finally, the remaining airline specific factors are captured through the θ_a parameter.

C. Extension II: Market Entry Deterrence

This second model is similar to the *basic model* except that the player decisions are now sequential rather than simultaneous. The idea of modeling the frequency competition as an extensive form game was proposed by Wei and Hansen [16] where, for contractual or historical reasons, one airline has the privilege of moving first, that is, deciding the frequency on a segment. The other airline responds upon observing the action by the first player. The *basic model* and the first extension implicitly assumed the existence of at least two competing airlines on a segment. However, frequency decisions in markets with only one existing airline are not completely immune to competition and the incumbent airline must account for the possibility of entry by another competitor while deciding the optimal frequency. Such situations can be modeled using the idea of Stackelberg equilibrium [21] or a sub-game perfect Nash equilibrium of an extensive form game. In this situation, the incumbent carrier is the Stackelberg leader and the potential entrant is the follower. A potential new entrant is denoted by a' . Inequality (3) can be extended as,

$$Q_{as} \leq \frac{f_{as}^{\alpha_s}}{f_{as}^{\alpha_s} + f_{a's}^{\alpha_s}} M_s \quad (9)$$

$$f_{a's} = \operatorname{argmax}_{f \in \mathbb{Z}^+} \left(\min \left(\frac{f^{\alpha_s}}{f_{as}^{\alpha_s} + f^{\alpha_s}} M_s, \right. \right. \\ \left. \left. LF_{\max} S_{a's} f \right) p_{a's} - C_{a's} f \right). \quad (10)$$

D. Solution Algorithm

Computation of a Nash equilibrium solution entails computing the frequency decisions of all the airlines such that the decision of each airline is optimal corresponding to the equilibrium decisions of all the other airlines. The computation of the optimal decisions by each airline, in itself, is a discrete optimization problem whose continuous relaxation is non-linear and non-convex. The total strategy space for a typical problem size is of the order of 10^{50} . To solve this problem, we propose a heuristic based on the idea of myopic best response, which employs successive optimizations, and individual optimization problems are solved to full optimality using a dynamic programming-based technique. For a full description of the solution algorithm, its computational performance and convergence properties, the reader is referred to [22].

IV. DATA AND MODEL VALIDATION

All the numerical results correspond to LaGuardia (LGA) airport, which has traditionally been one of the few slot controlled airports in the United States. The data on existing frequencies, average fares, aircraft sizes and segment passengers is obtained from the BTS website [4]. We conducted the validation as well as the numerical experiments described in section VI using data corresponding to a weekday in January 2008.

For all segments where only one carrier provides nonstop service, we use the market share function given by model extension 2. We use the market share function given by model extension 1 for segments on which: 1) the competitors' average fares differ by more than 5%; and/or 2) one or more major carriers operating a narrow- or a wide-body fleet compete against one or more regional carriers operating small jets. For all the other segments, we use the market share function given by inequality (3) in the *basic model*.

We validate the equilibrium frequencies predicted by the model using actual schedule data obtained from the BTS website [4]. We use data from LaGuardia airport at New York and compare the equilibrium frequencies predicted by our model against the actual values of frequencies. At LaGuardia airport, the maximum number of slots for each airline is restricted and each airline usually wants to make use of all the slots available to it in order to avoid losing any slots in the subsequent season. The minimum and maximum numbers of slots available to an airline, that is, L_a and U_a are assumed to be equal. Therefore, for our experiments, we assume that the total number of slots allocated to each airline is fixed. The airline needs only to decide the number of slots to allocate to flights to each of its destinations. Let f_{as} be the actual frequency of airline a on segment s and \widehat{f}_{as} be the equilibrium frequency as predicted by our model. The model ensures that the total frequency for each airline remains constant. Therefore, when the model overestimates the frequency on one segment it necessarily underestimates the frequency on some other segment corresponding to the same carrier. In order to avoid double counting of error, we define a measure of error particularly suitable for such situations. The mean absolute error (MAE) is defined as,

$$\text{MAE} = \frac{\sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}_a} \max(\widehat{f}_{as} - f_{as}, 0)}{\sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}_a} f_{as}}. \quad (11)$$

The actual frequency and the frequency estimated by our model for each carrier to each destination is presented in Fig. 1 on the x-axis and y-axis respectively. The overall MAE was found to be 7.2%. The model predictions thus match actual frequencies reasonably well.

V. SLOT ALLOCATION SCHEMES

In this section, we describe two simple strategies for allocating the available slots among different airlines and evaluate the performance of each strategy under the Nash equilibrium modeling framework.

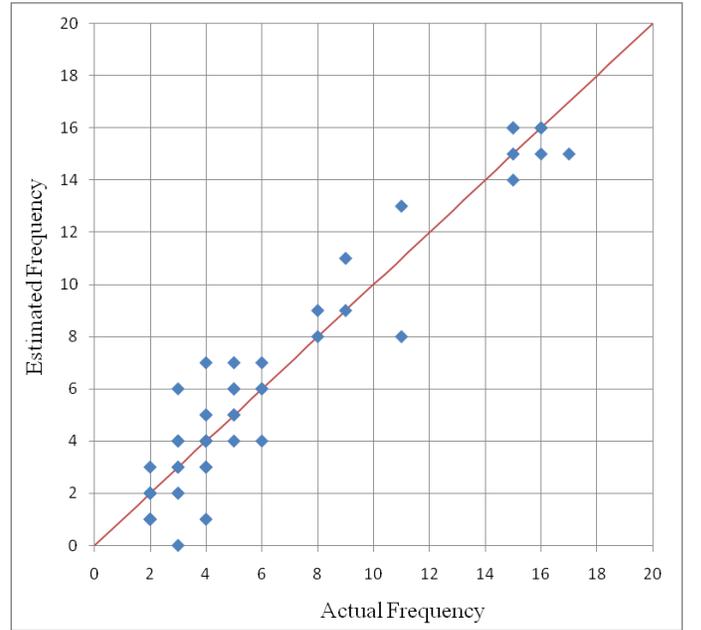


Figure 1. Empirical validation of frequency predictions.

A. Proportionate Allocation

Under the existing administrative controls, airlines often receive a similar number of slots from year to year. Historical precedent is usually used as the main criterion for slot allocation. There is opposition from the established carriers to any significant redistribution of slots. In the spirit of maintaining much of the status quo, our first slot distribution strategy involves proportionate allocation of slots. We vary the total number of slots at an airport while always distributing them among different carriers in the same ratio as that of actual flight schedules. For example, if the total number of slots at an airport is reduced from 100 to 80 and if the 100 slots were distributed as 40 and 60 between two carriers, then under our proportionate allocation scheme, the 80 slots will be distributed as 32 and 48 between the same two carriers.

B. Reward-based Allocation

While the proportionate allocation scheme is likely to be considered more acceptable by major carriers, it ignores the level of efficiency with which an airline utilizes its slots. Airlines differ, often substantially, in the number of passengers carried per flight or per slot. The idea behind the reward-based allocation is to reward those airlines which carry more passengers per slot, due to larger planes and/or higher load factors, and penalize those who carry fewer passengers per slot. Under this scheme, the number of slots allocated to each airline is proportional to the total number of passengers carried by that airline. In the previous example, if the first airline currently carries 140 passengers per slot and the second airline currently carries 120 passengers per slot, then under our reward-based allocation scheme, when the total number of slots is reduced to 80, the first airline will receive 35 slots and the second airline will receive 45 slots.

VI. NUMERICAL RESULTS

We performed two different experiments which are described in sub-sections A and B of this section. Sub-section C provides the sensitivity of the results to our assumption of the maximum load factor value. Sub-section D describes the impact of relaxing the constant aircraft sizes assumption.

A. Experiment I

In the first experiment, we varied the total number of allocated slots at LaGuardia and studied the impact on two important metrics, namely, the total operating profits of all the airlines and the total number of passengers carried. Fig. 2 and 3 show the change in total operating profits of all the airlines with slot reductions under the proportionate and reward-based allocation schemes, respectively. Fig. 4 and 5 show the change in the total number of passengers carried, assuming that the aircraft type (and seating capacity) for each airline on each segment remains unchanged upon slot reduction. The total number of passengers carried decreases as the number of slots decreases, but at a much lower rate. For the proportionate allocation scheme, up to a 35% slot reduction, each 1% reduction in slots leads to, on average, just a 0.38% reduction in the total passengers. A 35% reduction in slots leads to approximately 13% reduction in total passengers. Beyond 35%, each 1% reduction in slots leads to slightly over 1% reduction in total passengers. Also, the total operating profits for the proportionate allocation strategy increase with increasing slot reduction percentage up to 35%. Beyond that point, the operating profit starts to decrease. Very similar patterns are observed for the reward-based allocation strategy. Up to a 40% reduction in slots, each 1% reduction in slots leads to, on average, just a 0.27% reduction in the total passengers. A 40% slot reduction results in less than an 11% reduction in total passengers. However, beyond that point, the rate of reduction in total passengers is close to 1, similar to that in the proportionate reduction case. Similarly, total operating profit increases up to a 40% reduction and decreases thereafter.

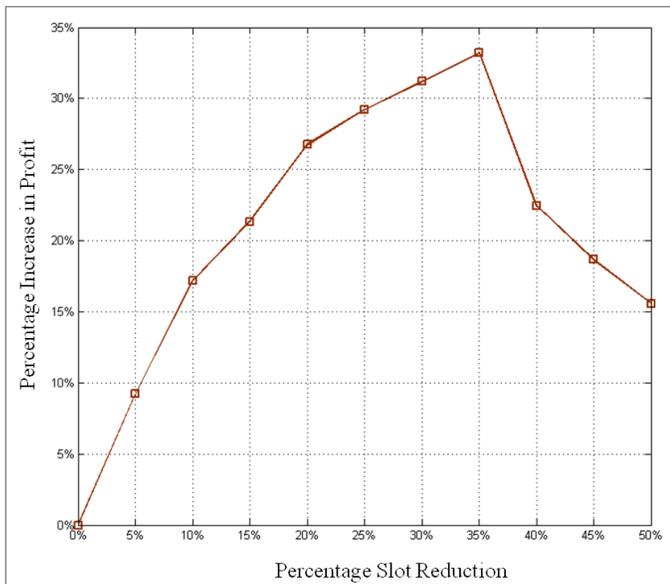


Figure 2. Total operating profit as a function of slot reductions under a proportionate allocation scheme assuming constant aircraft sizes.

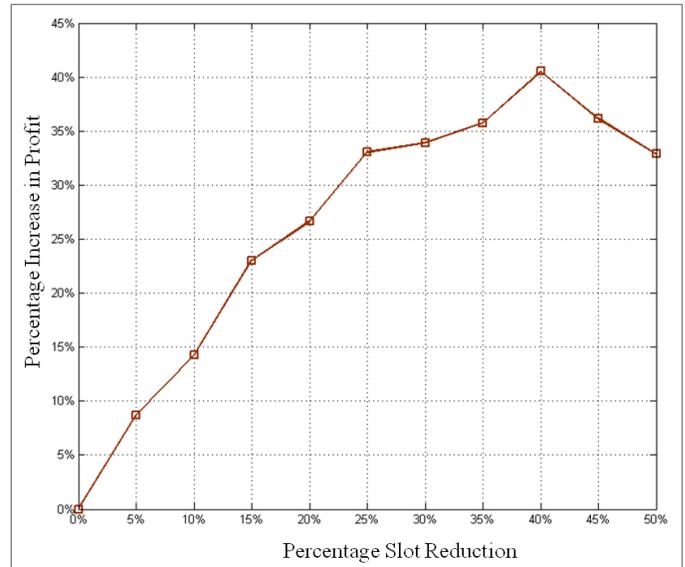


Figure 3. Total operating profit as a function of slot reductions under a reward-based allocation scheme assuming constant aircraft sizes.

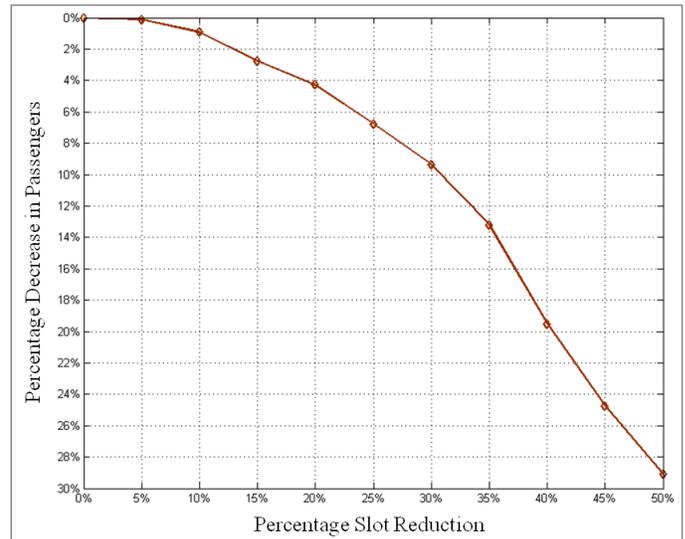


Figure 4. Total number of passengers carried as a function of slot reductions under a proportionate allocation scheme assuming constant aircraft sizes.

These effects are easy to understand intuitively. Given that aircraft sizes remain constant, the initial reduction in the number of slots results primarily in increases in load factors and hence, under our constant fares assumption, operating costs decrease at a faster rate than the rate of decrease in total revenue. So the operating profit increases. This effect continues until a point where the aircraft size constraint becomes binding and reduces the number of passengers almost proportionally to the number of slots. Therefore the operating revenue decreases at almost the same rate as the operating cost decrease, causing the operating profit to decrease. As the total number of slots decreases, the congestion and delays also decrease.

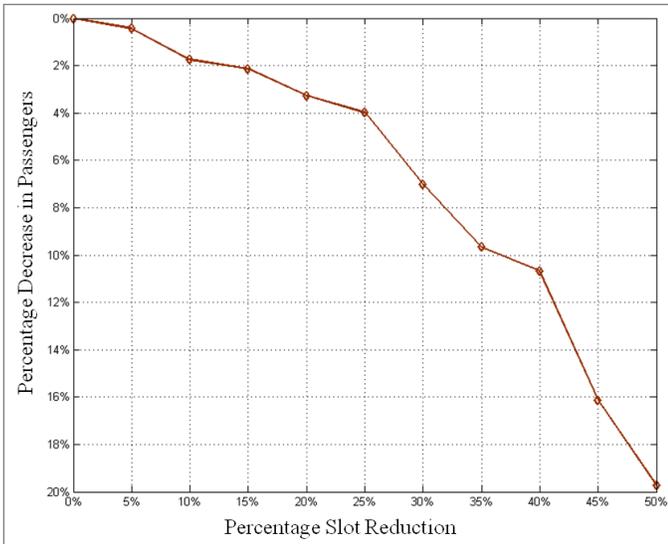


Figure 5. Total number of passengers carried as a function of slot reductions under a reward-based allocation scheme assuming constant aircraft sizes.

B. Experiment II

In our second experiment, we fixed a particular level of slot reduction and evaluated its system-wide impacts on the airlines (both individually and as a group), and on the passengers, based on multiple metrics. We considered the impact on the following metrics: airline operating profits, average flight delays, average passenger delays, total number of passengers carried, and average schedule displacement for passengers. The airport capacity benchmark report published by the Federal Aviation Administration (FAA) [23], sets the IFR (Instrument Flight Rules) capacity, that is, the bad-weather capacity, of LaGuardia airport at approximately 87.7% of its good-weather capacity. Currently, the number of operations scheduled at LaGuardia is close to the good-weather capacity. We chose to evaluate the case of a 12.3% reduction in slots, which approximately corresponds to scheduling at the IFR capacity instead of at the good-weather capacity.

Next, we describe the procedures used to estimate the average flight delays, the average passenger delays and the average schedule displacement. In order to estimate the impact on the average flight delays, we used the estimates of realized capacity values for an entire year (made available from Metron Aviation®) and actual flight delay data (obtained from the airline on-time performance database available on the BTS website [4]). While the number of operations currently scheduled at LaGuardia is close to its good-weather capacity, realized capacity drops to the IFR value during bad weather. Using the data on realized capacities and flight delays, the average delays to flights landing at LaGuardia are calculated for both good and bad weather conditions. We approximated the average flight delays under IFR capacity when the number of scheduled operations also equals the IFR value by the average flight delays under the good-weather capacity when the number of scheduled operations also equals the good-weather capacity value. After the 12.3% slot reduction, the average flight delays under the good weather conditions will be lower than those under good weather conditions without slot reduction. However, in order to be conservative in our delay

reduction estimates, we assumed that the average delays under good weather conditions remain unchanged upon slot reduction. Finally, we calculated the overall average flight delay as the expected value of delays under good- and bad-weather capacity.

In addition to flight delays, passenger itinerary disruptions due to flight cancellations and missed connections are responsible for a significant component of passenger delays. Reference [24] estimated that the ratio of average passenger delay to average flight delay in the domestic US for the entire 2007 year was 1.97. We used this value for computing the average passenger delays from the average flight delays.

The total trip time for the passengers is also affected by what is known as schedule displacement [6]. Schedule displacement is a measure of the difference between the time when a passenger wishes to travel and the actual time when he/she can travel given a flight schedule. The higher the daily frequency of flights, the lower is the schedule displacement. Due to slot reduction, the flight frequency on some segments is expected to reduce, which affects schedule displacement adversely. Schedule displacement is expressed as K/F , where F is the flight frequency and K is a constant which depends on the distribution of flight departure times and the distribution of desired times when passengers wish to travel. In this research, we will assume both these distributions to be uniform, which means that K equals $T/4$, where T is the time duration over which frequency F is distributed. Flight departures from LaGuardia are distributed between 6 am and 10 pm. So we will assume T to be 16 hours.

Table II summarizes the impacts of slot reduction to airlines and passengers based on various metrics. These results correspond to a 12.3% reduction in slots for both proportionate and reward-based allocation strategies, and the values in parentheses indicate the percentage change in each metric. The level of congestion depends on the total number of slots and not on the distribution of these slots among different airlines. Therefore, the delay reduction is the same under both proportionate and reward-based slot allocation strategies. Under either strategy, slot reductions lead to substantial reductions in average flight delays as well as average passenger delays. The total operating profits across of all carriers increase substantially. There is a small reduction in the total passengers carried. However, this is partly because we have assumed that aircraft sizes on each segment for each airline remain unchanged upon slot reduction. We will investigate the impact of relaxing this restriction partially in section 5.2. The average schedule displacement increases by just over 2 minutes. The total travel time for passengers departing from LaGuardia airport includes not only the schedule displacement and the duration of the flight out of LaGuardia, but also the airport access and egress times, and in cases of connecting passengers, the layover times and duration of the second flight. For flights out of LaGuardia airport, the average flight duration itself is 185.38 minutes. Therefore, in comparison, the increase in schedule displacement is negligibly small.

Table III presents the distribution of operating profits across different carriers. Again, the values in parentheses represent the percentage increases in profits. When the total number of slots

is reduced under either allocation strategy, the operating profit of each carrier increases compared to that under the no slot reduction scenario. The relative increase in operating profits is largest for the two regional carriers (RC 1 and RC 2) operating small regional jets out of LaGuardia airport. This is primarily because they had very low operating profit margins out of LaGuardia under the no slot reduction scenario. In fact, for one of regional carriers, the slot reduction helps achieve an operating profit instead of an operating loss, which is the case under the no slot reduction scenario. On the other hand, the network legacy carriers (NLC 1, NLC 2, NLC 3, NLC 4, NLC 5, and NLC 6) achieve the maximum absolute increase in operating profit per carrier. This is primarily because the average number of slots per day for network legacy carriers (36.83) itself is nearly 50% higher than that for the remaining carriers (24.50), and the average operating profit for the network legacy carriers per day (\$188,602) are much higher than that for the remaining carriers (\$26,501) under the no slot reduction scenario. The four low cost carriers are denoted by LCC 1, LCC 2, LCC 3, and LCC 4.

C. Sensitivity to the Maximum Load Factor Assumption

These results are obtained assuming a maximum average segment load factor (LF_{max}) of 85%. Now, we will present results on the sensitivity of slot reduction impacts to this assumption. We will focus on the sensitivity of the results of the second experiment. Table IV describes the sensitivity of total operating profits and total number of passengers carried to variations in the maximum average segment load factor value. Obviously, the average flight delays, average passenger delays and average schedule displacements do not change, because they depend only on the scheduled number of flight operations. The increase in total operating profit varies between 14.33% and 22.79%, and the decrease in number of passengers varies between 0.41% and 2.52%. Due to the integrality constraints on the number of slots, the results don't vary smoothly.

D. Impact of Limited Aircraft Upgauging

Results in experiments I and II were obtained under the assumption that, even when the total number of slots available to an airline is reduced, the airline will continue to operate the same-size aircraft as it did in the absence of slot reduction. This assumption might be realistic for very small reductions in the number of slots, but for significant reductions, it is reasonable to expect that the airlines will operate larger aircraft on some of the segments in order to accommodate more passengers and therefore increase profit. The main problem with modeling aircraft size decisions is that such decisions depend on the fleet availability. We estimate the impact of aircraft size upgauges by allowing for the possibility of a limited number of upgauges for each airline. We sort all the available types of aircraft operated out of LaGuardia by any of the airlines in increasing order of seating capacity. We allow a certain maximum percentage of an airline's fleet (operating out of LaGuardia airport) to be upgauged to the next bigger-sized aircraft. This constraint indirectly models the fact that an airline cannot arbitrarily increase aircraft sizes due to fleet availability constraints. We calculate the equilibrium frequency solution under the slot reduction scenario as described in section 5.1 and subsequently perform, for each airline, the most profitable

aircraft upgauges subject to the limits on maximum allowable upgauge percentage. As before, we assume a maximum average segment load factor of 85%.

Fig. 6 describes the impact of a limited number of aircraft upgauges on the reductions in total passengers when the total number of slots is reduced by 12.3%, and the proportionate allocation strategy is used. The maximum allowable upgauge percentage is on the x-axis, which represents the maximum percentage of an airline's flights that can be upgauged to the next bigger aircraft size. The percentage reduction in the total number of passengers varies from 2.27%, when no upgauges are allowed, to 0.88% when at most 20% upgauges are allowed for each airline. This shows that even with a small fraction of flights upgauging to a larger-sized aircraft, most of the reduction in the number of passengers disappears. The remaining reduction in the number of passengers is primarily attributable to the fact that there is only a limited number of aircraft sizes available; and on some segments, the number of passengers who are denied a seat due to a smaller aircraft size is not large enough to justify a profitable upgauge to the next bigger aircraft size.

TABLE II. EFFECT OF 12.3% SLOT REDUCTION

Metrics	No Reduction	12.3% Reduction	
		Proportionate	Reward-based
Total Operating Profits ^a	\$1,237,623	\$1,475,217 (19.20%)	\$1,446,520 (16.88%)
Avg. NAS Delay per Flight	12.74 min	7.52 min (-40.97%)	7.52 min (-40.97%)
Total Passengers Carried	22,184	21,680 (-2.27%)	21,728 (-2.05%)
Avg. Passenger Delay ^b	25.10 min	14.81 min (40.97%)	14.81 min (40.97%)
Avg. Schedule Displacement	25.35 min	27.58 min (8.8%)	27.55 min (8.7%)

a. Excluding flight delay costs. b. Due to NAS delays.

TABLE III. INCREASE IN OPERATING PROFITS DUE TO A 12.3% SLOT REDUCTION

Carriers	No Reduction	12.3% Reduction	
		Proportionate	Reward-based
NLC 1	\$366,592	\$416,322 (13.45%)	\$406,107 (10.67%)
LCC 1	\$48,061	\$59,507 (23.82%)	\$59,507 (23.82%)
NLC 2	\$65,996	\$74,466 (12.83%)	\$70,581 (6.95%)
NLC 3	\$196,215	\$252,231 (28.55%)	\$252,900 (28.89%)
LCC 2	\$39,694	\$46,632 (17.48%)	\$48,331 (21.76%)
RC 1	\$19,831	\$31,318 (57.92%)	\$29,831 (50.43%)
NLC 4	\$112,578	\$143,084 (27.10%)	\$130,316 (15.76%)
RC 2	-\$1,579	\$39,126 (n.a.)	\$40,582 (n.a.)
NLC 5	\$208,020	\$224,697 (8.02%)	\$218,922 (5.24%)
NLC 6	\$181,855	\$187,834 (3.29%)	\$189,443 (4.17%)

TABLE IV. SENSITIVITY OF SLOT REDUCTION IMPACTS TO THE MAXIMUM AVERAGE SEGMENT LOAD FACTOR VALUE UNDER A 12.3% SLOT REDUCTION

Maximum Average Segment Load Factor	Change in Total Profits		Change in Total Passengers Carried	
	Proportion ate	Reward-based	Proportion ate	Reward-based
75%	15.83%	14.33%	- 2.44%	- 2.23%
80%	17.39%	17.55%	- 2.52%	- 1.94%
85%	19.20%	16.88%	- 2.27%	- 2.05%
90%	22.79%	16.44%	- 0.41%	- 1.49%
95%	18.90%	17.59%	- 1.82%	- 0.94%

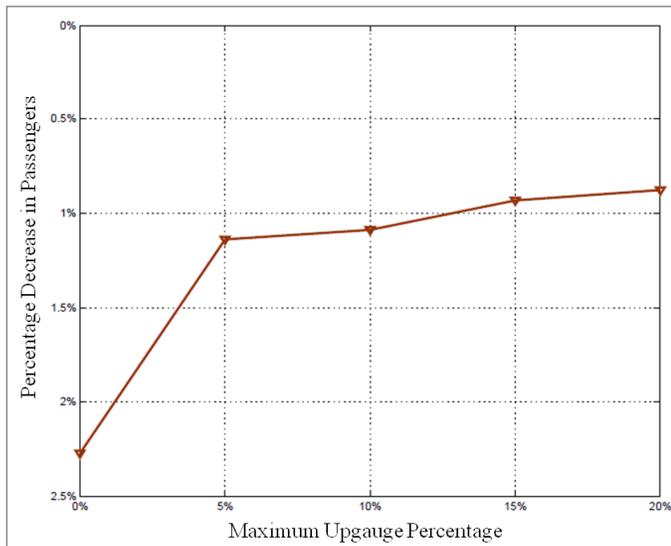


Figure 6. Effect of limited upgauging on total number of passengers under a 12.3% slot reduction.

VII. CONCLUSION

A slot control strategy involves deciding the (a) total capacity to be allocated and (b) the distribution of this capacity across different airlines. For a given slot distribution across airlines, each airline decides the frequency of flights in different markets with due consideration to competing carriers' frequency decisions. To the best of the authors' knowledge, this is the first study that tries to model airline competition under slot constraints. We developed a game theoretic model of airline frequency competition based on the S-curve, which is a popular model of market share in the airline literature. We justified the predictive power of the Nash equilibrium solution concept using empirical validation of the model estimates under existing slot allocation.

We evaluated two different slot reduction strategies. The results showed that in addition to a substantial reduction in flight and passenger delays, small reductions in total allocated capacity can improve the operating profits of carriers considerably. While the two strategies led to some differences in the actual profitability increases across individual carriers, the aggregate impacts were similar. Under each strategy, slot reduction led to a substantial increase in the profits of all

carriers across the board, and substantial reductions in flight delays and passenger delays. It also led to a small reduction in the number of passengers carried. However, most of the reduction in total passengers carried was eliminated when the possibility of a limited number of aircraft upgauges was introduced. The increase in schedule displacement due to the slot reduction was negligibly small compared to the overall travel times of the passenger.

Thus a small reduction in the total number of slots at congested airports is beneficial to the carriers, all of whom experience reductions in delay costs as well as increases in planned operating profits. It also benefits the passengers, almost all of whom get transported to their respective destinations, with negligible increases in schedule displacement and significantly lower average passenger delays. It is also beneficial to the airport operators because congestion and airport delays are reduced substantially. From the perspective of the entire system, slot reduction strategies lead to almost all passengers being transported with fewer flights, less delays, and lower total cost. Hence, slot reduction strategies are also attractive from the perspective of overall social welfare.

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